A Model of Stock Index Security Trading: Information, Volume and Pricing

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ABSTRACT

This paper develops a model of trading in stock and stock index security markets in the presence of transaction costs. We show that the introduction of stock index market improves the dissemination of market-wide information and index trading is more informative about stock market price movements than stock trading. The model generates rich implications on the informativeness of the stock index price, the causes and consequences of index arbitrage and the bi-directional lead-lag relation between the index and the stock markets. The implications of the model are tested using S&P 500 index options data.

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I. Introduction

Trading in stock index securities has grown dramatically in both scale and scope in the past two decades.[1] To analyze the trading activity and its implications in these seemingly redundant securities, at least three questions should be addressed: First, who will trade in the stock index securities? In other words, does the introduction of the index securities change the liquidity of the existing markets? Second, does the addition of the new securities improve the information transmission process in the financial markets overall? Or, does index trading provide additional price discovery in the financial markets? Third, how is the index security market integrated with the stock market? If security prices in all markets reflect all available information, then what are the mechanisms that facilitate information transmission across markets?

Despite considerable policy debates and extensive empirical studies on the effect of index derivative trading in the financial market, index derivative trading has received little theoretical analysis.[2] Exceptions are the work by Subrahmanyam (1991) and Gorton and Pennacchi (1993). Both papers study the function of the stock index as a trading venue for uninformed liquidity traders. They show that when a stock index contract is introduced, uninformed investors will migrate from the stock market to the index market, because adverse selection costs are lower in the stock index market than in the stock market. These models provide an explanation on the popularity of stock index securities. However, they do not study the issue of market integration and changes in the information transmission process when the stock index market is introduced. Because the trading behavior of the informed traders is symmetric in the stock index and the stock markets, index trading does not provide additional price discovery beyond the stock market. Furthermore, both papers

[1]The daily dollar trading volumes on S&P 500 index futures and options are \$25 billion and \$9 billion respectively. In contrast, the daily dollar stock trading volume on NYSE is \$30 billion. (All data are from exchanges press releases in 2000.) New index securities have been frequently introduced in various exchanges and by investment banks. Some of the more popular contracts include index shares (e.g., SPDRs, DIMONDS), sector index shares, index notes, and HOLDRs. Trading volume in SPDRs is larger than any other individual stocks in the U.S. market.

[2]Mayhew (1999) provides a nice survey on both theoretical models and empirical evidence about the impact of derivative securities on the spot market. While existing literature covers a broad range of topics, the central theme in these studies is whether the introduction of the derivative market affects the stability of the stock market.

imply that market integration is achieved through informed traders trading simultaneously in the index and the stock markets, other mechanisms such as index arbitrage do not exist.

In this paper we develop a model of stock index trading to examine its implications of price discovery and information transmission in the financial market. Our analysis complements the previous research that have explored the market liquidity implications of stock index trading. To capture features of cross markets trading, we assume that there exists one type of market friction, transaction costs. We formally derive the trading behavior of informed traders in the presence of transaction costs. We then use this basic setup to examine the implications of differently informed traders trading in and across different markets. Particularly, we study the effect of index trading on market liquidity, price informativeness, arbitrage opportunities, and the lead-lag relation in both the stock index market and the stock market.

To investigate how index trading affects market equilibrium in the stock and the stock index markets, we first study the trading decision of informed traders in the presence of transaction costs. Though informed traders are not restricted to trading in a specific market, we show that informed traders with systematic information will trade in the stock index but will not trade in all stocks. We also show that the incentive for traders with security specific information to trade in the index market is weak. Transaction costs have long been advocated as an important reason for stock index trading. We derive the impacts of one type of transaction costs on informational trading decisions.

Including transaction costs in the analysis allows us to examine the effect of index trading on the properties of security prices. The model provides an unified framework in explaining the price discovery and information transmission process in the financial market with index trading. Our analysis provides new insights on market price informativeness, arbitrage opportunities and the lead-lag relation in the stock and the stock index markets. Using a number of informativeness measures, we show that the price of the stock index in the index market conveys more systematic information than the price of the stock portfolio in the stock market. Thus, trading in stock index market provides additional information beyond information in the stock market. We further demonstrate that the lead-lag relation between the index and the stock markets is bi-directional. When systematic information is strong, index market leads stock market. When aggregate security specific information is strong, stock market leads index market. We establish that arbitrage opportunities arise because traders with different information trade in different markets. As a consequence, arbitrage activity not only is a mechanism of achieving market integration but also provides information on the fundamental security value not reflected in the current security prices. We also confirm the major conclusion in Subrahmanyam (1991) and Gorton and Pennacchi (1993) that the stock index market is the preferred venue for liquidity trading.

The theoretical results obtained in this paper are consistent with a broad range of empirical findings on the relation between index trading and stock market. The results also present some new testable implications. We specifically test the implication that index trading is more informative about stock market price movement than stock trading. We test the association between stock price changes and the trading volumes in the index market and the stock market. Using S&P 500 index options trading volumes and S&P 500 index stock trading volumes, we find that the relation between S&P 500 index option trading volumes and S&P 500 stock price movement is much stronger than the relation between S&P 500 index stock trading volumes and S&P 500 stock price movement. More importantly, S&P 500 index stock trading volumes provide no information beyond that contained in the S&P 500 index option trading volumes.

The paper is organized as follows. Section II summarizes empirical findings on stock index trading that motivate our theoretical analysis in this paper. Section III presents the model setup and derives the trading decision of informed traders in the presence of transaction costs. Section IV derives the market equilibrium condition with informed trading and examines its implications on market liquidity, price informativeness, arbitrage opportunities and the lead-lag relationships. Section V describes the empirical test, and Section VI concludes.

II. Empirical Evidence on Index Derivatives Trading

Since the introduction of index derivative securities, empirical studies have examined the effects of index derivative trading from various directions. In this section, we briefly review some of the results that represent the scope of current empirical studies, and in particular, the results that motivate our theoretical analysis. For simplicity, we classify the empirical studies into four categories: the information content in index trading, the effect of index trading on stock return properties, the lead-lag relation between the index market and the stock market, and the causes and consequences of index arbitrage.

A. Information in Index Trading

Many empirical studies have examined the motive or the information implications of index trading. Different interpretations on index trading volume or open interests have been offered by various authors. For example, Bessembinder, Chan and Seguin (1996) use stock index futures open interest as a proxy for divergence of traders' opinions. Chang, Chou and Nelling (2000), however, interpret stock index futures open interest as a proxy for hedging demand caused by past volatility. Bessembinder and Seguin (1992) find evidence that index futures trading represents informed trading. But Chen, Cuny and Haugen (1995) conclude that increasing open interest represents uninformed or liquidity trading. These empirical studies yield no consensus on the information content in index trading. The theoretical literature has provided little guidance on these different interpretations.

B. Index Trading and Stock Return Properties

Many observers believe that rapid dissemination of market-wide information through index derivative market has fundamentally altered the behavior of the underlying equity market. Consistent with this view, several papers have found strong evidence that properties of stock returns have changed subsequent to the introduction of index derivative securities. Froot and Perold (1995) show that the positive stock index autocorrelation found in earlier studies was a result of high autocorrelation during the 1960s and 1970s. The positive autocorrelation vanished completely by the later 1980s. After examining several alternative explanations, they attribute this to improved market-wide information dissemination because of index derivative trading. Antoniou, Holmes and Priestley (1998) find that the asymmetric stock return volatility phenomenon migrated from stock market to index futures market after the introduction of index futures trading, suggesting that index markets are more responsive to news arrivals.

C. The Lead-Lag relation

Another line of research has provided evidence on where informed traders trade by examining the lead-lag relation between index derivative market and the stock market. Kawaller, Koch and Koch (1987), Stoll and Whaley (1990), and Chan (1993) all find that index futures lead the stock market. However, the lead-lag relation between the index futures market and the stock market is not constant. Specifically, Chan (1993) shows that when more stocks move together (market-wide information) the futures leads the cash index to a greater degree. Chan (1993) interpret this as suggesting that the futures market is the main source of market-wide information. More recently, Frino, Walter, and West (2000) find that the index market lead the stock market when there is significant macroeconomic news and the stock market leads the index market around stock-specific information release. These results indicate that in addition to considering how fast one market reflects new information relative to the other market, it's equally important to investigate what new information one market incorporates relative to the other.

D. Causes and Consequences of Index Arbitrage

A long standing question in the index derivative literature is the role of index arbitrage. Though there are theoretical models explaining why arbitrage opportunities exists in equilibrium (see, for example, the clientele model of Holden (1995) and Chen, Cuny and Haugen (1995), and the information model of Kumar and Seppi (1994)), these models do not provide specific directions for empirical tests on the effect of index arbitrage trading activities. In particular, while Kumar and Seppi (1994) base their analysis on different information sets in the index and stock markets, the cause and nature of such information differential are not clear. Empirically, the properties of futures prices and "basis" have been examined extensively. For stock index futures, MacKinlay and Ramaswamy (1988) and Brennan and Schwartz (1990) indicate that observed future prices reflect active arbitrage activity. There are many papers examining the consequences of index trading, especially whether index arbitrage destabilizes the stock market. Few have investigated the causes of index arbitrage activity. The only paper we are aware of studying the information content in index arbitrage trading is Hasbrouck (1996), who finds that index arbitrage trading contains information useful for predicting stock price changes beyond the information already reflected in the index prices.

III. A General Model With Transaction Costs

In this section, we describe a general model with transaction costs and derive the results that informed traders endogenously determine their trading decision in the presence of transaction costs. The model is similar to Kyle (1985) and Subrahmanyam (1991). Our description of the economy closely follows the discussion in Subrahmanyam (1991) except that we here include transaction costs.

A. The General Economy

The economy is defined on a discrete time, finite horizon with a sequence of trading rounds at times $t = 1, ..., T$. In most part of the discussion, we only use a one-period trading model. We explicitly consider the multi-period trading activity when we examine the lead-lag relation between the index market and the stock market.

The economy consists of N risky assets and a stock index contract written on the index composed of the N risky assets (say, the S&P 500 index). The N securities are simultaneously traded in the spot market (henceforth the stock market) with their value governed by the following process:

$$S_{nt} = \overline{S}_{nt} + \beta_n \gamma_t + \epsilon_{nt}, \quad n = 1, \ldots, N \tag{1}$$

This is a "factor model" adopted by Subrahmanyam (1991). $\overline{S}_{nt}$ is the value of security n at time $t-1$, which is public information. γ is a vector of systematic factors and is called "factors" and ϵ_{nt} is the security specific or idiosyncratic components of the security value innovation. $\epsilon_{1t}, \epsilon_{2t}, ..., \epsilon_{nt}$ and γ are mutually independent and are assumed to be normally distributed with mean zero. The value of a stock index consisting of the N stocks is expressed as:

$$S_t = \sum_{n=1}^{N} w_n \overline{S}_{nt} + \sum_{n=1}^{N} w_n \beta_n \gamma_t + \sum_{n=1}^{N} w_n \epsilon_{nt} \tag{2}$$

Where w_n is the weight of security n in the index, such that $0 \leq w_n < 1$ and $\sum_{n=1}^{N} w_n = 1$. When weight w_n is zero, stock n is not included in the index. For simplicity, we assume that β_n and w_n are constant through the entire trading periods. The stock index is traded as a separate security. There are thus $N + 1$ markets for the N risky assets and the index. When the context is clear, we will omit the t subscript in the following discussions.

We do not distinguish between the stock index securities and stock index derivative securities. Stock index securities include index shares such as SPDRs, DIAMONDS and various types of HOLDRs. Stock index derivative securities include mainly index futures, index options and options on index futures. The "value" feature of the index derivative securities are inherently difficult to model (See Back (1993), Biais and Hillion (1994), Easley, O'Hara and Srinivas (1998), and Cao (1999) for models on individual stock derivative securities). We adopt the approach in Subrahmanyam (1991), Gorton and Pennacchi (1993) and Kumar and Seppi (1994) to model the "trading" feature of the derivative securities, i.e., the informational, hedging and liquidity trading purposes in the derivative market are different from those in the stock markets. The results derived here should apply to both stock index securities and stock index derivative securities.

B. Information and Trading

There are two types of investors in the economy whom we broadly refer to as informed and liquidity traders respectively. Informed traders trade to exploit superior information, liquidity traders trade for pure liquidity reasons or to hedge their risk exposures. In addition, there is a risk-neutral market maker for each of the markets. One group of informed traders has information on the common risk factors γ.[3] This group of informed traders is referred to as factor informed traders or traders with systematic information. Another group of informed traders has information on the idiosyncratic term ϵ_n for a particular security n. This group of informed traders is termed as security specific informed traders. For

[3]Private information on common factors are difficult to obtain. An alternative interpretation is that factor informed investors are those who are able to process public information faster and more efficiently than others are. See Admati and Pfleiderer (1988) for similar interpretation.

simplification, we also assume that informed traders observe the information perfectly.[4] Private information only lasts one trading period.

For the liquidity traders, we define one type as "hedgers" or "factor" liquidity traders. They trade in the index market or stock market for portfolio liquidation purposes or to hedge their risk exposures in the stock market. The term "hedgers" is rather loosely defined. It includes liquidity traders constrained to trade in the index market and liquidity traders who can choose to trade either in the index or in the stock market. The two types of hedgers can be interpreted as "non-discretionary" and "discretionary" in the terms of Admati and Pfleiderer (1988) and Subrahmanyam (1991). Another type of liquidity traders is the pure liquidity or noise traders who trade for exogenous reasons in the stock market. For simplicity, we use "hedgers" and "noise traders" to refer to the liquidity traders in the index and stock markets respectively. All traders are risk neutral.

Before deriving the results on informed traders' trading decision, we now discuss the assumption of transaction costs and its implications on trading behavior. One of the most important features of stock index trading is its comparative cost advantage over trading individual stocks. This cost advantage has been the major force behind the introduction of index securities in general, and index shares such as "SPDRs" and "DIAMONDS" in particular. In this paper, we explicitly take the effect of transaction costs on informed trading into account. For simplicity, we assume a symmetric fixed transaction cost structure. Under this structure, transaction costs are fixed and are the same for all stocks and the stock index. This definition of transaction cost can be interpreted as representing the set-up cost of trading in each security.[5] With transaction costs, informed traders' decision on whether to trade in one security depends on whether expected profit in trading the security is larger than the transaction costs incurred.

[4] As shown in many earlier studies, adding noise signal may not add too much insight but complicates the exposition considerably. For the purpose of the current paper, the simple perfect information structure is adequate. Results using noisy information are similar and can be requested from the author.

[5] See Cho, Shin and Singh (1999), Coval and Hirshleifer (2000) for similar discussions on set-up transaction costs. Even though transaction cost is same for a single stock and the stock index, trading in a stock portfolio is more expensive than trading in the stock index. Thus the simple fixed cost structure captures the essential difference of transaction costs in trading stocks and the stock index.

C. Trading Decisions with Transaction Costs

Without transaction costs, factor informed traders and security specific informed traders will trade in both the index and stock market. Without cross-markets order flow observations, the availability of the stock index market is just an additional trading opportunity for the factor informed traders. For security specific informed traders, possessing information about a stock is equivalent to possessing noisy information about the index if the stock is a component in the index. To examine the effect of transaction costs on informational trading decision, we first present the market equilibrium results with no transaction costs.

For stock n, market maker sets the security price equal the expected value conditional on the aggregate net trading order ω_n in equilibrium.

$$P_{nt} = E(S_{nt}|\omega_n) \tag{3}$$

The market maker sets the price by a linear pricing rule:

$$P_{nt} = \overline{S}_{nt} + \lambda_n \omega_n \tag{4}$$

where λ_n is called the "pricing parameter" and the inverse of λ_n is a measure of "liquidity" of the market for security n.

Define the total number of factor informed traders as g. We assume that all the g factor informed traders observe perfectly the systematic factors γ. Similarly, the number of informed traders with security specific information is k_n for security n, and they also all observe perfectly the security specific information ϵ_n. The liquidity trading in the index and stock markets is assumed to be fixed in this section. For security n in the stock market, the following lemma gives the results of market equilibrium with no transaction costs. (Proofs of Lemmas and Propositions are in the Appendix unless otherwise stated.)

Lemma 1 *Without transaction cost, the factor informed trader i submits order*

$$x_{in} = \frac{\beta_n \gamma}{(g+1)\lambda_n} \quad ,$$

and the security specific informed trader trader j submits order

$$x_{jn} = \frac{\epsilon_n}{(k_n+1)\lambda_n} \quad ,$$

where λ_n *is given by*

$$\lambda_n = \sqrt{\frac{k_n var(\epsilon_n)}{(k_n+1)^2 var(z_n)} + \frac{g\beta_n^2 var(\gamma)}{(g+1)^2 var(z_n)}} \quad ,$$

and z_n *is the total liquidity trade for stock* n *in the stock market.*

For the stock index security, we obtain similar results in the following Lemma.

Lemma 2 *The factor informed trader* i *submits order*

$$x_i = \frac{\sum_{n=1}^{N} w_n \beta_n \gamma}{(g+1)\lambda} \quad ,$$

and the security specific informed trader j *submits order*

$$x_j = \frac{w_n \epsilon_n}{(k_n+1)\lambda} \quad ,$$

where λ *is given by*

$$\lambda = \sqrt{\sum_{n=1}^{N} w_n^2 \frac{k_n var(\epsilon_n)}{(k_n+1)^2 var(z)} + \frac{g(\sum_{n=1}^{N} w_n \beta_n)^2 var(\gamma)}{(g+1)^2 var(z)}} \quad ,$$

and z *is the total liquidity trade for the index security in the index market.*

Without transaction costs, factor informed traders and security specific informed traders will trade in both the index and stock markets. With transaction costs, the trading decision of informed traders depends on whether the expected profit given the information endowment is larger than the transaction costs incurred. We now derive the profits before transaction costs for both the factor informed and security specific informed traders.

Lemma 3 *The profits before transaction costs for each factor informed trader and each security specific informed trader in trading stock* n *are given by:*

$$\pi_{n,g} = \frac{\beta_n^2 var(\gamma)}{(g+1)^2 \sqrt{\frac{k_n var(\epsilon_n)}{(k_n+1)^2 var(z_n)} + \frac{g\beta_n^2 var(\gamma)}{(g+1)^2 var(z_n)}}} \tag{5}$$

$$\pi_{n,k_n} = \frac{var(\epsilon_n)}{(k_n+1)^2 \sqrt{\frac{k_n var(\epsilon_n)}{(k_n+1)^2 var(z_n)} + \frac{g\beta_n^2 var(\gamma)}{(g+1)^2 var(z_n)}}} \tag{6}$$

Note that for factor informed traders, the *total* trading profit in the stock market is the sum of trading profits in all N stocks.

Similarly, the profits for the factor informed traders and the security specific informed traders trading in the stock index market are given by the following:

Lemma 4 *The profits before transaction costs for each factor informed trader and each security specific informed trader in the stock index market are given by:*

$$\pi_g = \frac{(\sum_{n=1}^{N} w_n \beta_n)^2 var(\gamma)}{(g+1)^2 \sqrt{\sum_{n=1}^{N} w_n^2 \frac{k_n var(\epsilon_n)}{(k_n+1)^2 var(z)} + \frac{g(\sum_{n=1}^{N} w_n \beta_n)^2 var(\gamma)}{(g+1)^2 var(z)}}} \quad (7)$$

$$\pi_{k_n} = \frac{w_n^2 var(\epsilon_n)}{(k_n+1)^2 \sqrt{\sum_{n=1}^{N} w_n^2 \frac{k_n var(\epsilon_n)}{(k_n+1)^2 var(z)} + \frac{g(\sum_{n=1}^{N} w_n \beta_n)^2 var(\gamma)}{(g+1)^2 var(z)}}} \quad (8)$$

The trading profits of the informed traders are a function of the number of both types of informed traders and the payoff structure of the security. Several insights can be obtained from examining the above profit functions. First, in both the stock and index markets, the profits of security specific informed traders increase with the number of factor informed traders. It nevertheless is lower than with no factor informed trading. For the factor informed traders, higher security specific trading increases factor trading profits. The above result is due to the fact that the systematic factors and security idiosyncratic components are assumed to be independent. An increase in the number of one type of informed traders intensifies the competition between traders of the same type, thus benefiting the other type of informed traders. Second, in the stock market, the trading profits of factor informed traders increase in β_n, $var(\gamma)$, and of course the liquidity trading for that particular stock $var(z_n)$. But the profits decrease with the stock specific information $var(\epsilon)$. For security specific informed traders, the profits increase with $var(\epsilon)$ and $var(z_n)$, and decrease with β_n, $var(\gamma)$.

Denote the transaction costs as T_c for all stocks and the index. We now consider the trading decisions of the informed traders based on the profit function and the transaction cost. For the factor informed traders to trade in stock n and the stock index market, the following conditions should hold: $\pi_g \geq T_c$ and $\pi_{n,g} \geq T_c$. Especially, the threshold condition of at one factor informed trader trade in the stock or index market should hold, i.e., $\pi_1 \geq T_c$

, $\pi_{n,1} \geq T_c$. From factor informed traders' stock trading profit function, we can see that the possibility of factor informed traders trading in a stock increases with β_n and $var(z_n)$, but decreases with $var(\epsilon_n)$. In particular, if $\beta_n \rightarrow 0$, $\pi_{n,1} \rightarrow 0$, then $\pi_{n,1} < T_c$ holds for a constant T_c. Similarly, if the liquidity trading in stock n is low, i.e., $var(z_n) \rightarrow 0$, then $\pi_{n,1} \rightarrow 0$, and $\pi_{n,1} < T_c$ holds. In both cases factor informed traders do not trade in this stock. Thus stocks with low factor sensitivity (β_n), low liquidity trading ($var(z_n)$), or high security specific information ($var(\epsilon_n)$) may not attract factor informed trading. In general, certain types of stocks (e.g., small stocks) posses all three properties. This indicates that there exists strong possibility that the factor informed traders do not trade in *all* stocks.

Further observations can be obtained by comparing factor informed trader's profit function for trading stock n (Equation (5)) and trading the stock index (Equation (7)). Three components are different for the two profit functions: the factor sensitivities β's, the security specific components, and liquidity trading. In general, it is a reasonable assumption that the factor sensitivity of the index $\sum_{n=1}^{N} w_n\beta_n$ is fairly strong. Even if the factor sensitivities are the same, it is easy to see that the effect of the stock specific components is much smaller in the stock index because of the diversification effect. Thus, the profit is higher for factor informed traders in the stock index market than in a comparable stock market. The above analysis implies that the possibility of factor informed traders trading in the index market is stronger than the possibility of trading in individual stocks.

Similar arguments apply to the trading decision of the security specific informed traders. Because this type of traders are informed with security specific information only, they will trade in that stock if the profit from trading is higher than the transaction costs, i.e., $\pi_{n,k_n} \geq T_c$. Traders with security specific information on stock n do not trade in other stocks on which they do not have information. By comparing the two profit functions for security specific informed traders to trade the stock and the stock index, we can see that the possibility of security specific informed traders trading in the stock index is much lower than trading in the individual stocks.

Now consider the trading decision by the security specific informed traders to trade in the stock index. The possibility of security specific traders trading in the stock index increases with w_n and $var(\epsilon_n)$. Assume the security specific innovation has finite variance, i.e., $var(\epsilon_n)$ is bounded by $\overline{\sigma}^2$. In a well diversified stock index, when $w_n \rightarrow 0$, π_{n,k_1}

converges to 0. Thus all traders informed with security specific information do not trade in the stock index. This result is intuitive, a well diversified index does not attract security specific informed trading. A weaker result is that, even with less well diversified stock index, security specific informed traders with information on low w_n stocks do not trade in the stock index. It is possible that when the stock index is not well diversified, security specific informed traders with information on some stocks will trade in the index market if the condition $\pi_{k_1} \geq T_c$ is satisfied. However, in a well diversified stock index, the above analysis indicates that the possibility for security specific traders to trade in the index market is low.

The results on cross market informational trading decision in the presence of trading costs are summarized in the following proposition.

Proposition 1 *Under a constant transaction costs structure, the trading decisions of informed investors trade across markets are determined by their information endowments, the valuation structure of the security and the transaction costs. In particular, when $\pi_{n,1} < T_c$ for some stock n, factor informed traders do not trade in those stocks. Similarly, in a well diversified stock index, $\pi_{k_1} < T_c$ holds and security specific informed traders do not trade in the index market.*

The above conclusion is a direct result of the transaction costs assumption. It provide a starting point for analyzing the effect of informed trading on market equilibrium and security price properties in the index and the stock markets. In the above analysis we assume that liquidity trading in the index market and stock market is predetermined. As we show in the next section, index market is the preferred trading place for liquidity traders. In general, this result strengthens the conclusion in this section.

IV. Market Equilibrium and Implications

Under the assumed simple symmetric transaction costs structure, informed traders determine their trading decision endogenously. As a result, differently informed traders trade in different markets and may not trade across the index and stock markets. These informational trading decisions affect the market equilibrium condition and have great impact on the properties of security prices in the stock and stock index markets. In this section, we study the market equilibrium condition with transaction costs, and investigate the implications on market liquidity, price informativeness, the arbitrage opportunities, and the lead-lag relation in the index and stock markets.

A. Market Equilibrium with Cross-market Trading

To examine the effect of informational trading decision on the properties of security prices, we first present the market equilibrium condition with cross-market informed trading in the presence of transaction costs.

Motivated by the results from last section, we assume that factor informed traders will not trade in all stocks and security specific informed traders do not trade in the stock index.[6] This simplification enables us to derive easily interpretable results. More specifically, factor informed traders only trade in a subset of individual stocks. The number of this set of stocks is M, with $M < N$. From the properties of the profit functions in the last section, it is easy to see that the M stocks are not a random sample from the individual stock universe. Whether factor informed traders trade in a specific stock is endogenously determined by the stock's payoff structure and liquidity trading activity. Compared with the $N - M$ stocks that factor informed traders do not trade in, the M stocks have higher $\beta's$, higher liquidity trading, and lower security specific value innovation.

Because only the factor informed traders trade in the index market, the demand function and the equilibrium price schedule are different from the results for index trading in *Lemma* 1 with no transaction costs. When factor informed traders trade in stocks m ($m = 1, 2, ..., M$) in the stock market, the trading order of the factor informed traders and the security specific informed traders will have the same format as in *Lemma* 1. If factor informed traders do not trade in stocks n ($n = 1, 2, ..., N - M$), the results are obtained with only security specific informed trading. The following results describe the market equilibrium condition for both the index and the stock market with informed trading endogenously determined.

[6] A more general assumption is that factor informed traders trade in a subset of individual stocks, but a subset of security specific informed traders, $k_n, n = 1, ..., L, L < N$ trade in the index market. Results using this assumption are qualitatively similar to the current results, but nevertheless more complicated.

Lemma 5 *The equilibrium pricing parameter in the stock index is given by*

$$\lambda = \frac{1}{g+1}\sqrt{\frac{g\beta^2 var(\gamma)}{var(z)}} \quad ,$$

The equilibrium pricing parameters in stock n with and without factor informed trading are given by

$$\lambda_n = \sqrt{\frac{k_n var(\epsilon_n)}{(k_n+1)^2 var(z_n)} + \frac{g\beta_n^2 var(\gamma)}{(g+1)^2 var(z_n)}} \quad ,$$

$$\lambda_n{}' = \frac{1}{k_n+1}\sqrt{\frac{k_n var(\epsilon_n)}{var(z_n)}} \quad .$$

Where λ_n and $\lambda_n{}'$ are for with and without factor trading respectively, and z and z_n are the total liquidity trading for the index and stock n. We use $\beta \equiv \sum_{n=1}^{N} w_n\beta_n$ for notation simplicity.

B. Market Liquidity

Subrahmanyam (1991) and Gorton and Pennacchi (1993) show that the index market is the preferred trading venue for uninformed liquidity traders because adverse selection costs are lower in the index market than in the stock market. Their models do not consider the effect of transaction costs by both the informed traders and liquidity traders. With transaction costs, liquidity traders trading in the individual stocks would incur higher transaction costs than trading in the stock index. However, this does not imply that liquidity traders prefer to trade in the stock index market. The reason rests on the fact that liquidity traders face another type of costs, i.e., the loss to the informed trading in the respective markets. In principle, "discretionary" liquidity traders will trade in either the index market or the stock market to minimize the sum of the transaction costs and the losses to informed trading. Without examining the loss to informed trading, it is unclear whether or not liquidity traders would prefer to trade in the index market.

There are both advantages and disadvantages for hedgers to trade in the stock market. The advantage is the liquidity trading by noise traders for each security in the stock market which increases the total liquidity trading. The disadvantage is the presence of both factor informed and security specific informed traders in the stock market. When trading in the index market, hedgers only face factor informed traders. As a consequence, the condition here for hedgers to trade only in the index market is different from that in Subrahmanyam (1991) even without considering the transaction costs by liquidity traders. We now formally show that the change of informed trading in the presence of transaction costs strengthens the condition under which discretionary liquidity traders only trade in the index market.

Consider the simple case where factor informed traders trade in the index and all the underlying stocks and security specific informed traders only trade in the stock market.[7] Define the demand of discretionary hedgers as z_h, and the noise trading in security n as z'_n. Discretionary hedgers will trade only in the index market if the loss to informed traders in index market is lower than the sum of losses to informed traders in the stock market. More formally, the following condition holds:

Proposition 2 *When factor informed traders trade in both the stock market and the index market and security specific informed traders only trade in the stock market, discretionary liquidity traders only trade in the index market if the following condition is satisfied:*

$$\sum_{n=1}^{N} w_n \sqrt{\Big[\frac{var(\epsilon_n)}{K_n} + \frac{\beta_n^2 var(\gamma)}{G}\Big][var(z_{hn}) + var(z'_n)]^{-1}} > \sqrt{\frac{\beta^2 var(\gamma)}{G}[var(z)]^{-1}} \qquad (9)$$

In the above equation, we define $K_n = \frac{(k_n+1)^2}{k_n}$ and $G = \frac{(g+1)^2}{g}$ to simplify the notation. z is the total liquidity trading in the index market which includes both discretionary and non-discretionary liquidity trading. The above condition is stronger than the condition presented in Proposition 2 in Subrahmanyam (1991). He shows that with all security specific informed traders trading in the index, the tendency for the discretionary liquidity traders to trade in the index is strong. The main reason for his conclusion is that index provides a diversification benefit for the security specific information, liquidity trader's loss to security specific informed traders is minimized in the index market. In our case, because security specific informed investors do not trade in the index, there is *no* loss to security specific informed traders in the index market. Hence, the tendency for the liquidity trading property to hold is stronger.

With our assumed symmetric transaction cost structure, the above condition is more likely to hold. Clearly, hedgers incur higher costs in trading in portfolio of individual stocks.

[7]Assuming factor informed traders only trade in a subset of stocks does not qualitatively change the results.

The above result confirms the major conclusion in Subrahmanyam (1991) and Gorton and Pennacchi (1993) that the stock index market is the preferred venue for liquidity trading.

C. Price Informativeness

What's the effect on price discovery in the financial market when the index market is introduced? Does index trading provide additional price discovery in the financial market? The behavior of security prices and trading volumes in the index and the stock markets are determined by the respective informed trading in the two markets. In this subsection we examine the relative price informativeness in the index and the stock market.

Given the assumed factor structure, it is easy to see that the informativeness of the stock price increases with the addition of factor informed traders trading in the stock market. Define the noisiness of price as the variation of the deviations of prices from their true value, $Q_n \equiv var(S_n - P_n)$. Thus, the inverse of Q_n represents the informativeness of prices. The following gives the noisiness of security n price with and without factor informed trading.

$$Q_n = \frac{var(\epsilon_n)}{k_n + 1} + \frac{\beta_n^2 var(\gamma)}{(g+1)}$$

$$Q'_n = \frac{var(\epsilon_n)}{k_n + 1} + \beta_n^2 var(\gamma)$$

Where Q_n (when factor informed traders trade in the stock market) is always smaller than Q'_n (when factor informed traders do not trade in the stock market).

Another measure of informativeness, the reduction in the variance of prices, $I_n \equiv var(S_n) - var(S_n|P_n)$, measures the extent to which prices reveal private information. Not surprisingly, this measure and the noisiness measure are closely related. The measures on the reduction in price variance with and without factor informed trading are given as follows.

$$I_n = \frac{k_n var(\epsilon_n)}{k_n + 1} + \frac{g\beta_n^2 var(\gamma)}{(g+1)}$$

$$I'_n = \frac{k_n var(\epsilon_n)}{k_n + 1}$$

Clearly, when price variation reduction is used as an informativeness measure, price with factor informed trading is more informative.

We are more interested in the relative informativeness of the index price and the price of the individual stock portfolio. The noisiness of the index price and the price of the stock portfolio are defined as $Q \equiv var(S-P)$ and $Q' \equiv var(\sum_{n=1}^{N} w_n S_n - \sum_{n=1}^{N} w_n P_n)$. The values are given in the following proposition:

Proposition 3 *Under the factor structure with cross markets trading, the noisiness of the index price and the price of the stock portfolio*

$$Q = \sum_{n=1}^{N} w_n^2 var(\epsilon_n) + \frac{(\sum_{n=1}^{N} w_n \beta_n)^2 var(\gamma)}{(g+1)} \quad , \tag{10}$$

$$\begin{aligned} Q' &= \sum_{n=1}^{N} w_n^2 \frac{var(\epsilon_n)}{k_n+1} + var(\gamma)\Big((\sum_{n=1}^{N} w_n \beta_n)^2 - \frac{g(\sum_{m=1}^{M} w_m \beta_m)^2}{(g+1)}\Big) \\ &+ \frac{g var(\gamma)}{(g+1)^2}\Big(\sum_{m=1}^{M} w_m^2 \beta_m^2 - (\sum_{m=1}^{M} w_m \beta_m)^2\Big) \quad . \end{aligned} \tag{11}$$

The above proposition gives the two noisiness measures for the index (Q) and the portfolio of individual stocks (Q').

The index price does not incorporate security specific information, thus Q contains a larger term on the idiosyncratic components than Q' as represented by the first term of both equations. One plausible implication is that the price of the index is less informative than the price of the portfolio of individual stocks on the security specific information. Especially when idiosyncratic information components in the portfolio is strong, i.e., when $\sum_{n=1}^{N} w_n^2 var^2(\epsilon_n)$ is large. If the sum of the idiosyncratic components approaches zero in a well diversified stock portfolio, it may be justified to obtain $\sum_{n=1}^{N} w_n^2 var^2(\epsilon_n) \to 0$ when $N \to \infty$ in Q and Q'. In this case, the informativeness of the stock portfolio will converge to the informativeness of the stock index on the security specific components.

The price of the portfolio of stocks only reflects factor information in M of the N stocks. This is one reason that index price is more informative on the systematic factors than the portfolio of individual stocks. From the above noisiness measures, first examine the second term on the RHS in equation (12). $(\sum_{n=1}^{N} w_n \beta_n)^2$ is always larger than $(\sum_{m=1}^{M} w_m \beta_m)^2$ when the β's are of the same sign. In this case, the second term in equation (12) is always larger than the second term in equation (11). This implies that index price is more informative on the systematic factors. Even if the β's are not of the same sign, we would expect this inequality to hold if systematic factors affects most stocks in the same direction. This certainly is a reasonable assumption. The third term in equation (12) is of ambiguous sign. Note that, $(\sum_{m=1}^{M} w_m \beta_m)^2$ may be larger than or smaller than $\sum_{m=1}^{M} w_m^2 \beta_m^2$, depending on the signs of β's. One scenario is that when all the β's are of the same sign, the third term is negative and the informativeness of the portfolio of individual stocks on the systematic factors increases. However, even in this case, the effect of the third term is still dominated by the effect of the second term. As a result, one expects the property that the informativeness on the factor information is stronger in the index market than in the stock market to hold generally.

The above result on price informativeness is different from that in Subrahmanyam (1991). He shows that the informativeness of the index price is the same as the price of the portfolio of stocks on the idiosyncratic components. When the β's are of the same sign, price of the stock portfolio is more informative on the systematic factors than the index price (see Proposition 5 in Subrahmanyam (1991)). The difference results from that Subrahmanyam (1991) does not have the second term in equation (12). When all the β's are of the same sign, price of stock portfolio is more informative on the systematic factors than the index price because of the third term. In our results, even when all the β's are of the same sign, this effect is still dominated by the second term in equation (12). This result is intuitive and is further tested in the next section.

D. Arbitrage Opportunities

Index arbitrage is widely viewed as one major mechanism to achieve market integration between the index derivative market and the stock market. In practice, the effect of index arbitrage on the stock market movements is controversial. To understand the effect of index arbitrage activity, it is important to examine why arbitrage opportunities exist in the first place. This subsection investigates the existence of price differentials between the index and the stock markets. By studying the properties of arbitrage opportunities, we provide an explanation of the causes and consequences of arbitrage activity in a unified framework of informed trading.

Because security specific informed traders do not trade in the index market, arbitrage opportunities exist even when factor informed traders trade in both the stock and index market.[8] When factor informed traders can only trade in M individual stocks, arbitrage opportunities reflect mispricing on both the factor innovations and the security specific value innovations.

Define the "basis" B as the difference between the price of portfolio of individual stocks and the index price. Let $B \equiv P - \sum_{n=1}^{N} w_n P_n$ denote the basis, the following proposition gives the value of "basis".

Proposition 4 *Under the factor structure with cross markets trading, the basis is given by the following:*

$$\begin{aligned} B &= \lambda\omega - \sum_{n=1}^{N} w_n \lambda_n \omega_n = \lambda z - \sum_{m=1}^{M} w_m^2 \lambda_m z_m - \sum_{n=1}^{N-M} w_n^2 \lambda_n{}' z_n \\ &+ \frac{g\gamma}{g+1}\left(\sum_{n=1}^{N} w_n \beta_n - \sum_{m=1}^{M} w_m \beta_m\right) + \sum_{n=1}^{N} w_n \frac{k_n \epsilon_n}{k_n + 1} \end{aligned} \tag{12}$$

The three pricing parameters (λ's) are from Lemma 6, with λ, λ_m and λ_n' present the pricing parameters for the stock index, stocks without factor informed trading and stocks with informed trading respectively.

The basis is a measure of market mispricing between the index and stock market. First, because the stock index does not incorporate stock specific information, arbitrage opportunities exist owning to the existence of security specific value innovations. This part is represented by the last term in the above "basis" formula. Note that the security specific value innovation can be negative or positive. Thus, it is not necessarily the case that price of stock portfolio is higher than the index price when security specific information is strong. Second, in general, the index price is more informative about the systematic information

[8]To make the model tractable, we do not explicitly define index arbitrageurs throughout the paper. Heuristically, the role of index arbitrageurs can be understood in the same framework as in Kumar and Seppi (1994) where index arbitrageurs do not have new information but can observe pricing differentials between index and stock market and have access to both markets. So pricing differentials between the two markets generates index arbitrage activity. The unconditional expected price differential is bounded by the transaction costs. It's important to notice that informed traders can not carry out arbitrage activities because the market clearing prices in all the markets are unknown when they submit their orders.

than the price of the portfolio of stocks. So arbitrage opportunities based on systematic information also exist. This is represented by the fourth term in the above formula. Finally, the first three terms in equation (12) consist of liquidity trading components which do not contain fundamental value innovations on either systematic factors or security specific components.

When there exists large imbalance in factor informed trading and security specific informed trading in the index and the stock market, index arbitrage opportunities exist. Specifically, when systematic information is strong, arbitrage opportunities are generated through the imbalance in factor informed trading across the index and the stock market. When aggregate security specific information is strong, arbitrage opportunities are generated through the security informed trading in the stock market.

Clearly, the price difference between the index and the stock market does contain information. This is a result that differently informed investors trade in and across different markets. Different trading activity has been mentioned in some papers as a possible reason for market mispricing, thus a possible reason for the existence of index arbitrage activity (See, for example, MacKinlay and Ramaswamy (1988)). However, no formal argument has previously been presented. Our results imply that index arbitrage or program trading activity should contain information not reflected in the current market price, as the empirical evidence in Hasbrouck (1996) shows. Another testable implication is that arbitrage activity will mostly occur when there is large imbalance between factor information and security specific information. Furthermore, this result may partly explain the puzzling evidence in Bakshi, Cao and Chen (2000) that call prices and the underlying stock do not always move in the same direction in the S&P 500 index options market.

A brief discussion of arbitrage activity is in order. It is possible that even when arbitrage opportunities exist, arbitrage activity may not occur because of transaction costs. This implies that arbitrage activity is carried out by arbitrageurs who can observe the price difference between the two markets and who have low transaction costs. Even with high transaction costs, one side arbitrage activity such as buying or selling the index contract is possible if fundamental information will be revealed in later trading periods.

E. Lead-lag Relation

Lead-lag relation between index and stock market has attracted considerable interests from empirical studies. Lead-lag relation between price movements of stock index and the underlying stocks reveal how fast one market reflects new information relative to the other, or, what new information one market incorporates relative to the other. In this subsection, we develop the existence of such relation in the framework of cross-market trading.

To analyze the lead-lag relation, we adopt a multi-period framework as presented in equation (1). This framework has essentially the same property as the one period model. Private information is useful for only one period. All liquidity trades, systematic factors and security specific innovations are serially uncorrelated. In addition, the variance of systematic factors and the variance of the security specific innovations are constant through all the trading periods.

Define the correlation coefficient as the the measure of lead-lag relation. The two measures for the two markets are as follows:

$$\rho_{I,t} \equiv cov\Big(\Delta \sum_{n=1}^{N} w_n P_{n,t}, \Delta P_{t-1}\Big)[var(\Delta P_{t-1})]^{-1/2}\Big[var\Big(\Delta \sum_{n=1}^{N} w_n P_{n,t-1}\Big)\Big]^{-1/2}$$

$$\rho_{S,t} \equiv cov\Big(\Delta P_t, \Delta \sum_{n=1}^{N} w_n P_{n,t-1}\Big)[var(\Delta P_{t-1})]^{-1/2}\Big[var\Big(\Delta \sum_{n=1}^{N} w_n P_{n,t-1}\Big)\Big]^{-1/2}$$

Where Δ denotes price changes from the previous period, and P, P_n are index price and stock n price respectively. Because the denominators are the same in the above expression, we can evaluate the lead-lag relation from one market to the other by comparing the two covariance measures.

The following Proposition provides equilibrium solution on the lead-lag measures.

Proposition 5 *The lead-lag relation can be presented using the following measures:*

$$cov\Big(\Delta P_t, \Delta \sum_{n=1}^{N} w_n P_{n,t-1}\Big) = \frac{g(\sum_{m=1}^{M} w_m \beta_m)^2 var(\gamma)}{(g+1)^2} + \frac{\sum_{n=1}^{N} w_n^2 var(\epsilon_n)}{k_n + 1} \tag{13}$$

$$\begin{aligned} cov\Big(\Delta \sum_{n=1}^{N} w_n P_{n,t}, \Delta P_{t-1}\Big) &= \frac{g(\sum_{n=1}^{N} w_n \beta_n)^2 var(\gamma)}{(g+1)^2} \\ &+ \frac{g^2 var(\gamma)}{(g+1)^2}\Big((\sum_{n=1}^{N} w_n \beta_n)^2 - (\sum_{m=1}^{M} w_m \beta_m)^2\Big) \end{aligned} \tag{14}$$

The above result presents clear prediction on the lead-lag relation between the two markets. From equation (14), stock market always leads the index in security specific information. For systematic information, in general, $(\sum_{n=1}^{N} w_n\beta_n)^2 > (\sum_{m=1}^{M} w_m\beta_m)^2$ holds, and the index market leads stock market. Thus the lead-lag effect exists for opposite direction. For empirical purposes, which effect dominates depends on the relative strength of factor innovation and security specific value innovation. When systematic information is strong, index market leads stock market; When aggregate security specific information is strong, stock market leads index market.

This result is consistent with the empirical findings in lead-lag relation between the stock index and the stock markets. Many studies find that the index derivative market leads stock market in price innovations (Kawaller, Koch and Koch (1987), Stoll and Whaley (1990), Chan (1993)). However, this lead-lag relation is not constant over time. In particular, Chan (1993) shows that when more stocks move together (market wide information), future market leads stock market to a greater degree. More recently, Frino, Walter, and West (2000) find that index market lead stock market when there is significant macroeconomic news, but stock market leads index market around stock-specific information releases. Our theoretical results show that the bi-directional lead-lag relation exists because the index market and the stock market incorporate different information at different speed.

Lead-lag relation is not inconsistent with arbitrage activities. The two are rather closely related in our discussion. Informed trading in different markets generates both arbitrage opportunities and lead-lag relation. Arbitrage opportunities represent contemporaneous price differences between the two markets, while lead-lag relation represent price changes with one period lag. It is possible that arbitrage activity reduces the magnitude of lead-lag relation, but may not eliminate this relation completely because of limited ability of identifying arbitrage opportunities and transaction costs. On the other hand, lead-lag relation must be constrained by the arbitrage activity.

V. An Empirical Test Using Index Trading Data

In the last section we present the main implications of an index trading model. The predictions of the model are broadly consistent with a wide range of empirical findings on index trading, arbitrage activities, and lead-lag relation. In this section, we provide further empirical evidence that supports one of the main predictions of the model.

On the informativeness of market price, the model shows that price of the stock portfolio is less informative on the systematic factors than the index price. The main reason is that factor informed traders only trade in a subset of stocks while security specific informed traders only trade in the stock market. If trading in the stock market is characterized by both factor informed trading and trading for security specific information, then stock trading will be less related with the stock index price changes or systematic factor innovations than index trading. Furthermore, with a well diversified stock portfolio, stock trading volumes are less related with changes in stock portfolio prices than index trading volumes.[9]

We specifically test the implication that index trading is more informative than stock trading about stock market price movement.[10] We examine the association between stock price changes and the trading volumes in the index market and the stock market. Bessembinder and Seguin (1992) find that the association between trading volume and stock market volatility decreases after the introduction of stock index futures contracts. They do not elaborate on the causes of such change. Using S&P 500 index option trading volumes and S&P 500 index stock trading volumes, we find that the relation between S&P 500 index option trading volumes and S&P 500 index price movement is much stronger than the relation between S&P 500 index stock trading volumes and S&P 500 index price movement. More importantly, S&P 500 index stock trading volumes provide no information beyond that contained in the S&P 500 option trading volumes. This finding indicates that index trading is more informative on stock market price movements. This result complements the findings in Bessembinder and Seguin (1992) and suggests that the volatility-volume relation migrates from stock trading activity to index trading activity.

[9]It is difficult to define trading volumes using net order flows. Admati and Pfleiderer (1988) use the square root of order flow variance as a proxy for trading volume. With this proxy we can formally show that the aggregate stock trading volumes are noisier than index trading volumes on the stock price movements. The intuition behind this result is that the idiosyncratic components vanish in a well diversified stock portfolio, but the trading volumes generated by idiosyncratic components information do not converge to zero.

[10]Further implications can be obtained by examining the lead-lag relation, the autocorrelation properties of stock prices conditional on the relative trading activity in the index and the stock market. Empirical tests on these implications are pursued in a separate paper.

A. Data and Empirical Methods

We use the S&P 500 index options traded on CBOE in our empirical tests. S&P 500 Index Option (SPX) is among the most actively traded financial derivatives in the world. Data on S&P 500 index options are provided by Berkeley Options Data Base (BODB). BODB provides complete information for every option transaction, including the time of the transaction, expiration month, strike price, transaction price, transaction size, and the underlying stock price as of the trade closest to the option trade. We use price and trading data on every S&P 500 index option traded during 1993. Using one-year's data, we do not need to control for the secular trends in both stock and option trading volumes. Another reason for using SPX is that the underlying index is representative of the U.S. stock market, and the stock market trading data for the stock index is readily available.

In this test, we develop three measures on market price movements based on both stock and option prices.[11] The three price variation measures are change in stock prices, change in option prices and change in implied volatility. All three measures can be proxies for market price variations even though the first one is the most widely used measure on stock market price movement. The three measures together provide a robustness check on our results.

We use the call and put prices of the most actively traded contract to calculate option price changes. Because the close hours are different for CBOE and NYSE and other major exchanges where S&P 500 stocks are traded, we delete all the data points after 3:00 PM CT. We calculate the call and put option returns separately. For each day we identify the first and the last trade on call and put options and use the trade price to calculate the option returns using *(last transaction price - first transaction price)/first transaction price.* Option market provides another measure for gauging changes in investor expectations. Change of the implied volatility represents change of the assessment on market condition independent of stock price changes. By using changes in implied volatilities, we focus on option price movements independent of changes in stock prices. We construct the implied volatility measure using the standard Black-Scholes formula. From the most actively traded contract, we take the first and the last observed call prices each day and compute the implied volatility

[11] For detailed discussion on the construction of market price movement and trading volumes measures, and the related empirical methods and results, see Wang (1999).

by numerically solving the Black-Scholes call option formula. We take the difference between the two as the measure of change in implied volatility. The last measure on market price variation is change in stock prices. To be consistent with data on option price movements, we use the S&P 500 stock index daily open to close returns.

For measures on trading volumes, we use both trading volumes and number of transactions. For the index option, the trading volume is the total number of contracts traded each day. Stock trading data on S&P 500 index is obtained from DataStream. They also include total number of stocks traded each day and total number of transactions.

We use two regressions to examine the association between market price changes and the trading volumes of stock index options and stocks. The first regression is

$$|\Delta P_t| = \alpha + \alpha_E E_t + \beta_V V_t + \epsilon_t \tag{15}$$

Where ΔP_t are the market price movement measures and V_t is the measure on trading activity. E_t is a dummy variable for the expiration effect in index option trading, which equals 1 when the near term contract has five days or less to maturity. This is a standard procedure that examines the relation between price changes and trading volumes in financial markets (See Karpoff (1987)).

The second regression is

$$|\Delta P_t| = \alpha + \alpha_E E_t + \beta_{VS} VS_t + \beta_{VO} VO_t + \epsilon_t \tag{16}$$

Where VS_t is the measure on stock trading activity and VO_t is the measure on index trading activity. This regression is used to examine the relative importance of stock trading activity and index trading activity in explaining market price movements.

B. Empirical Results

We use both trading volumes and number of transactions in the test. Because the two results are similar, we only report the results on trading volumes. The expiration date dummy has virtually no effect on the results and is not reported.

Results for regression one are reported in Table 1. Both index option trading volumes and stock trading volumes are positively correlated with market price movements. In addition to their own market, index trading volumes have explanatory power for stock price movements and stock trading volumes have explanatory power for index option movement measures. All the T-statistics are significant at the 1% level. However, the adjusted R^2s are uniformly smaller for the regression with stock trading volumes as the independent variable. This indicates that the explanatory power of the stock trading volumes is not as strong as that of the index trading volumes.

Results from regression two provide more convincing evidence. These results are reported in Table 2. When stock trading volumes and index option trading volumes are used together to explain market price movements, stock trading volumes are always not significant. The most notable evidence in Table 2 is that stock trading volumes have virtually no marginal explanatory power when index trading volumes are also used as an explanatory variable. This inference can be further verified by comparing results in Table 1 and Table 2. The coefficient estimates for index trading volumes change very little from Table 1 to Table 2. More strikingly, the R^2s remain virtually the same whether index trading volume is the only explanatory variable or both the index trading volume and the stock trading volume are used as explanatory variables. Consistent with this result, the correlation coefficient between stock trading volumes and index trading volumes is 0.42, not as high as ordinarily suspected.

Results from the two regressions are consistent with the prediction in the model that index trading is more informative on market price movements. In studying the relation between equity option trading and individual stock trading, Back (1993) predicts that option prices will be more sensitive to its own market trading activity and stock price variations will be more sensitive to stock trading than to option trading. Our results show that for index trading, index option trading volumes are more informative on both stock market movements and option market movements than stock trading volumes. This evidence may result from the “index” feature rather than the “option” feature in the index option trading activity.

VI. Conclusion

This paper develops a model of trading in the stock and the stock index security markets in the presence of transaction costs. After deriving the trading behavior of informed traders in the presence of transaction costs, we examine the effect of index trading on market liquidity, price informativeness, arbitrage opportunities, the lead-lag relation. Empirical tests based on the model's prediction are performed.

We study the effect of index trading on price discovery and information transmission in a unified framework. We show that the introduction of stock index market improves the dissemination of market-wide information and index trading volume is more informative about stock market price movements than aggregate stock trading volume. The model further generates rich implications on the informativeness of index price, the causes and consequences of index arbitrage and the bi-directional lead-lag relation between index and stock market.

The predictions of the model are broadly consistent with recent empirical findings. For example, recent studies show considerable informed trading in the index options market (Nofsinger and Prucyk (1999)). In some studies, index trading volumes are more informative than stock trading volumes market (Wang (1999)). The results of the model and the related evidence suggest that index trading activity would be better viewed as informed trading associated with systematic factors.

The results support the conjecture and empirical results in Chan (1993) and Frino, Walter, and West (2000) on the lead-lag relation between the index and stock markets. More importantly, the results provide new predictions and/or explanations on both old and new trading practices in the market places. We show that arbitrage opportunities arise because of the different informational trading in the two markets. When either factor or security specific informed trading clusters in one market, differences in market prices are created. Furthermore, the current model provides sharper predictions on both arbitrage trading activity and the information content in arbitrage trading. While prediction on arbitrage trading activity is yet to be tested, the prediction on the information implication explains why arbitrage trading is informative as documented in Hasbrouck (1996).

The basic results and implications obtained in this paper provide a new angle to analyze the behavior of closed-end fund discounts. The results can explain several important parts of the discount puzzle, especially the evidence on the mean reversion of the discount and the discount's predicting power over small stock returns. The factor structure assumption and the asymmetric trading structure developed in this paper can be applied in other areas as well. In most multi-market trading mechanisms, we often observe "related" securities traded cross markets, rather than the exactly same security traded across markets (A case analyzed in Chowdhry and Nanda (1991) and others). For example, in the pre-hour trading market, only index securities and a number of highly liquid large stocks are actively traded. If a factor structure is assumed for the index and all the stocks, the framework in this paper can be used to examine the trading decision of the informed traders and the information transmission across trading sessions. In an international setting, if we assume a global factor structure, the model can be extended to explain the transmission of stock returns, volatilities, volumes and especially the contagion effect across international stock markets.

Appendix

Proof of Lemmas and Propositions:

Proof of Lemma 1: Factor informed trader i observes perfectly the systematic factor γ. Security specific informed trader j observes perfectly the security specific information ϵ_n for stock n. γ and ϵ_n are independent. For stock n, the informed trader's objective function is:

$$E(x_{n,l}(S_n - P_n)|I_l)$$

Where l represents i and j and I_l is the information set for trader l.
Let trader i conjecture that other factor informed traders submit order $A\beta_n\gamma$. Denoting the order by each security specific informed trader as $x_{n,j}$. Then the factor informed trader i's objective function is:

$$E(x_{n,i}(\beta_n\gamma + \epsilon_n) - x_{n,i}\lambda_n(x_{n,i} + (g-1)A\beta_n\gamma + kx_{n,j} + z_n)|\gamma)$$

This is maximized when:

$$x_{n,i} = \frac{\beta_n\gamma - (g-1)A\beta_n\gamma}{2\lambda_n}$$

Set $x_{n,i} = A\beta\gamma$, we get

$$x_{n,i} = \frac{\beta_n\gamma}{(g+1)\lambda_n}$$

Similarly, for security specific informed trader j, the order flow is:

$$x_{jn} = \frac{\epsilon_n}{(k_n+1)\lambda_n}$$

To get the pricing parameter λ_n, note that $\lambda_n = Cov(\beta_n\lambda + \epsilon_n, \omega_n)/Var(\omega_n)$, where ω_n is the total order flow in the market for stock n. Substituting $x_{n,i}$ and $x_{n,j}$ in the above expression, we get:

$$\lambda_n = \sqrt{\frac{k_n var(\epsilon_n)}{(k_n+1)^2 var(z_n)} + \frac{g\beta_n^2 var(\gamma)}{(g+1)^2 var(z_n)}}$$

Where z_n is the total liquidity trading for stock n.

Proof of Lemma 2: See proof of Lemma 1.

Proof of Lemma 3 and Lemma 4: The profit function for informed trader l for stock n is given by $\pi_{n,l} = E(\lambda_n\omega_n x_l)$. The profit function for informed trader l for the stock index is given by $\pi_l = E(\lambda \omega x_l)$. Substituting the pricing parameters λ_n, λ and the order flows in the stock and stock index markets in Lemma 1 and 2 gives the results in Lemma 3 and 4.

Proof of Lemma 5: See proof of Lemma 1.

Proof of Proposition 2: The pricing parameters λ_n, λ for stock n and the stock index are given in Lemma 1 and 2. Applying the condition $\sum_{n=1}^{N} w_n \lambda_n > \lambda$ yields Equation (10).

Proof of Proposition 3: Define $Q \equiv var(S - P)$ and $Q' \equiv var(\sum_{n=1}^{N} w_n S_n - \sum_{n=1}^{N} w_n P_n)$ as the noisiness measure for the index and the portfolio of individual stocks. With $P = \overline{S}_t + \lambda \omega$ and $P_{nt} = \overline{S}_{nt} + \lambda_n \omega_n$, substituting the pricing parameters λ_n, λ and the order flows in the stock and stock index markets gives the result.
Because security specific informed traders do not trade in the index market, the pricing parameter λ and the order flow in the index market are given in Lemma 5. Factor informed traders trade in M of the N stocks. The pricing parameters λ_m and the order flows for the M stocks are given in Lemma 1. The pricing parameters λ_n and the order flows for the $N - M$ stocks that factor informed traders do not trade in are given in Lemma 5.

Proof of Proposition 4: The "Basis" is defined as $B \equiv P - \sum_{n=1}^{N} w_n P_n$. The result can be obtained by substituting the pricing parameters λ_n, λ and the order flows in the stock and stock index markets in the above definition. See Proof of Proposition 3 for the specifications of pricing parameters and order flows.

Proof of Proposition 5: The result can be obtained by substituting the pricing parameters λ_n, λ and the order flows in the stock and stock index markets in the two covariance measures. See Proof of Proposition 3 for the specifications of pricing parameters and order flows.

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Table 1

Regression of Daily Market Price Movement on Trading Volumes.

Estimates of daily market price movement on daily trading volumes. We use call return, put return, change in implied volatility, and stock return as measures of market price movements. VS_t is S&P 500 index stock trading volumes and VO_t is S&P 500 index options trading volumes. We estimate regression (1) in the text:

$$|\Delta P_t| = \alpha + \alpha_E E_t + \beta_V V_t + \epsilon_t$$

Where ΔP_t are the market price movement measures and V_t is the trading volumes. E_t is an expiration date dummy. T-statistics are in parenthesis.

	VO_t	R^2	VS_t	R^2
Call Return	1.3	0.19	2.37	0.045
	(6.02)		(3.55)	
Put Return	0.9	0.17	2.00	0.034
	(4.33)		(3.09)	
Imp Volatility	0.5	0.23	1.23	0.023
	(3.82)		(2.59)	
Stock Return	0.18	0.21	0.31	0.080
	(8.21)		(4.70)	

Table 2

Regression of Daily Market Price Movement on Both Stock and Index Trading Volumes.

Estimates of daily market price movement on both stock and index trading volumes. We use call return, put return, change in implied volatility, and stock return as measures of market price movements. VS_t is S&P 500 index stock trading volumes and VO_t is S&P 500 index options trading volumes. We estimate regression (2) in the text:

$$|\Delta P_t| = \alpha + \alpha_E E_t + \beta_{VS} VS_t + \beta_{VO} VO_t + \epsilon_t$$

Where ΔP_t are the market price movement measures and E_t is an expiration date dummy. T-statistics are in parenthesis.

	VO_t	VS_t	R^2
Call Return	1.17	0.09	0.19
	(4.82)	(1.39)	
Put Return	0.76	0.11	0.17
	(3.21)	(1.63)	
Imp Volatility	0.47	0.07	0.24
	(2.82)	(1.43)	
Stock Return	0.16	0.012	0.22
	(6.62)	(1.78)	