

Information, Liquidity, and Asset Trading in a Random Matching Game*

HUGO A. HOPENHAYN

*University of Rochester, Rochester, NY 14627-0156; and
Universitat Pompeu-Fabra, Balmes 132, 08008 Barcelona, Spain*

AND

INGRID M. WERNER

Stanford Business School, Stanford, California 94305-5015

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This paper develops a sequential random matching model of asset trading to analyze how the extent of information about an asset that is available in the market can affect its tradeability. Liquidity traders are rational agents with higher impatience, which make optimal intertemporal consumption decisions given the trading constraints. Information asymmetries result in unexecuted trades. Agents who want to consume relatively early optimally choose to exchange initial assets for new assets that have lower expected payoff but are more liquid in subsequent trading. These assets have a lower expected rate or return (i.e., a liquidity premium) and higher trading volume. *Journal of Economic Literature* Classification Numbers: D83, G11, G14. © 1996 Academic Press, Inc.

1. INTRODUCTION

The central question in asset pricing theory is what characteristics of an asset are valued. The standard approach is to derive asset prices as the subjective value of future stochastic dividend streams. If agents held assets indefinitely, or traded them at no cost, this would follow from individual rational choice. In practice, agents hold many assets as store of value anticipating that they might have to be traded in the future. As a conse-

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quence, the value of an asset must also depend on how easily it can be traded: its liquidity.¹ Understanding the characteristics that make some assets more tradeable than others is thus important for asset pricing theory. This paper studies how the liquidity of assets may depend on the extent to which market participants are informed about their future payoffs.

We model trading as a sequential random matching game following [9, 16]. We study a three-period world where heterogeneity of preferences, captured by different rates of time preference, provides an incentive to trade. Risk neutral agents are endowed with assets in the initial period. All assets have the same ex ante probability of paying a dividend in period two versus period three. The assets are heterogeneous in two respects; the expected payoffs differ across assets, and the extent of information available about each asset varies. It is useful to think of the second characteristic as a measure of how easily the expected payoffs of an asset can be verified. A security which is hard to verify could, for example, be one which is in small supply, is traded infrequently, or for which not much information is disclosed in financial reports and through public announcements. Opportunities for trades arise as a result of random matching. The owner of an asset knows its expected payoff, but the trading partner might be uninformed. To the extent that agents are reluctant to trade unless they know the quality of an asset, this feature introduces a friction in the trading environment.

We provide conditions under which information makes an asset more tradeable. As a consequence, agents are willing to hold these assets even if they have a lower rate of return. In a series of papers, [2–4] provide empirical support for the hypothesis that liquidity affects asset pricing. They argue that one can approximate the value loss due to illiquidity by assessing the discounted value of expected future transactions costs. More liquid assets should have a lower expected return all else equal; i.e., “investors are willing to pay a yield concession for the option to liquidate their holdings before maturity at lower costs” [4, p. 1417]. It is this kind of option that gives additional value to assets in our model. Specifically, since relatively impatient agents might want to sell assets before maturity they value an asset more highly if it can be sold more easily in the second period. In our world, this corresponds to an asset for which information asymmetries are less likely to arise.

The literature on rational expectations has also emphasized the additional costs borne by liquidity traders as a result of information asym-

¹ Reference [10] suggests that market liquidity can be measured by the “‘tightness’ (the cost of turning around a position over a short period of time), ‘depth’ (the size of an order flow innovation required to change prices a given amount), and ‘resiliency’ (the speed with which prices recover from a random, uninformative shock)” of a market (p. 1316).

metries.² This cost arises since uninformed traders (or intermediaries) need to be compensated for the losses that may arise from the opportunistic behavior of traders that are informed. The result is larger bid-ask spreads for assets where information asymmetries are more pervasive. This makes such assets less attractive as store of value for agents with liquidity needs.³

An analogous effect occurs in our model. Liquidity traders (our impatient agents) potentially face uninformed traders. This is costly in that trades may not occur when such an information asymmetry arises. However, when the counterpart in the transaction is informed and trades do occur, the terms of trade are independent of the verifiability of the asset. Hence, though we share the focus on the role that *asymmetric* information has for increasing transactions costs, our transaction costs exclusively affect the ability of executing trades. The probability of being able to execute a trade is generally higher for assets whose future payoffs the market is more informed about. For thinly traded securities there simply might not be someone available who is willing to take the other side.

Recent models by [6, 7] show that liquidity traders, if permitted, choose assets or contracts with low costs from asymmetric information.⁴ Although our model has a very different structure, it shares the feature that agents with larger liquidity needs exhibit a stronger preference for assets where the information asymmetry is less pervasive, the *verifiable assets*. We explicitly model an initial stage of exchange by which assets are traded in such a way as to increase the weight of verifiable assets in the aggregate portfolio of impatient agents. As a consequence, verifiable assets play the role of store of liquidity and are thus more heavily traded.⁵ Since impatient agents are willing to pay a premium for future liquidity services, these assets also exhibit a lower (shadow) rate of return. This is consistent with the inverse relationship between volume traded and returns found in the data.

The decentralized trading environment described above makes trades quite difficult. Two ways of improving the trading technology are explored. The first one involves introducing a mechanism for ex ante investment in verifiability. The second one consists of introducing more centralized trading mechanisms which circumvent the matching technology and facilitate intertemporal trade. As in [6, 7], the intermediated mechanisms we study reduce transactions costs due to asymmetric information. In addition,

² A summary of the literature is given in [1]. Reference [5] is a recent paper that addresses the issue of disclosure and liquidity in the context of a market microstructure model.

³ An alternative way of capturing liquidity is exemplified by [15] which looks at the relationship among (fixed) transactions costs, liquidity, and market participation.

⁴ Optimal financial contracts are also affected by the presence of informed trading in [8].

⁵ A similar notion of liquidity is captured by one equilibrium in [9], which has the feature that agents sometimes trade a lower for a higher storage-costs commodity because they believe this to be more marketable in the future.

however, they reduce the frictions in the trading environment due to the double-coincidence-of-needs requirement for trades. The market structures we define involve verification of assets, i.e., monitoring, which makes participation costly. A decentralized market will therefore generally remain in existence and only a subset of the agents participate in the centralized market. The choice between search and centralized trade was explored by [13]. It was shown there that traders who expect to execute large trades will choose the decentralized market. We instead characterize which types of assets will be traded in a decentralized fashion and which ones will participate in more centralized exchange.

The paper proceeds as follows. Section 2 describes the economic environment. Two types of equilibria are derived and analyzed in Section 3. In the first one, assets which are more widely known in the market have a higher value. Such assets are also traded more frequently. A second type of equilibrium may arise where assets which are relatively unknown are considered more valuable. This follows from an incentive to hide the quality of assets if these can be traded without verification. Section 4 discusses the welfare implications of the ex ante stock market with liquidity trading. We analyze ways of reducing the frictions in the trading environment due to asymmetric information and random matching by introducing intermediated market mechanisms in Section 5. Summary and conclusions are given in Section 6.

2. ECONOMIC ENVIRONMENT

Consider the following three-period economy. The market has a continuum of three-period-lived agents with unit mass. There are two types of agents, patient and impatient, who are distinguished by their rate of time discount. Preferences for an agent of type i are given by

$$E_0 \left\{ \sum_{t=2}^3 \beta_t^i C_t^i \right\}, \quad (2.1)$$

where $0 < \beta_I < \beta_P < 1$ are the rates of time discount for impatient and patient agents respectively, and C_t^i is consumption of agent i in period t . Agents only consume once and then leave the market. A fraction μ_P of the agents are patient and a fraction $\mu_I = (1 - \mu_P)$ are impatient. Type is observable by everyone. This simplifies the analysis without qualitatively affecting our results.

In the first period, each agent is endowed with an asset, let us call it a tree. Each tree is indivisible and distinguished by two features; the quality of the tree, and the extent to which that quality is observable upon inspec-

tion. Trees can either give a large payoff (L) or a small payoff (S). Payoffs are indivisible, and can usefully be thought of as a fruit of different size.⁶ Quality is measured by the probability, q , that the tree gives a large payoff (L). For future reference, let us define the expected payoff from a tree as $\pi(q) = S + q(L - S)$. Verifiability of a tree is parameterized by the probability, n , that the quality of tree (q, n) is observed upon inspection. With probability $(1 - n)$, the quality is not observed upon inspection. Should a trade still take place, the quality of the tree is revealed to the buyer after the trade is completed. The verifiability of a tree is observable to everyone. For each agent, the type (q, n) of the tree he is endowed with is drawn independently from a common distribution $F(q, n)$.

Assumption 1. F is continuous for all n .

Agents are asymmetrically informed about the quality of each tree; whereas the owner of the tree has perfect information, potential buyers may not be informed about the quality of the tree at the time of purchase. The verifiability of a particular tree measures how informed potential buyers are about the quality of that tree, i.e., the extent of asymmetry of information for that specific tree. Verifiability can usefully be thought of as the class of asset to which the tree belongs. Since the quality of a tree is always revealed after trading, at the extremes assets whose quality is unobservable, $n = 0$, are pure experience goods while those which are always observed, $n = 1$, are pure search goods as defined [11, 12].

Trees give fruit in the second period with probability δ and all trees that did not give fruit in the second period do so in period three. The size of fruit carried by an agent is observable. Though in the pool of assets, some are ex post more long lived than others, they have ex ante the same expected maturity. The time to maturity is independent of the other characteristics of the asset: its quality and verifiability. In consequence, no ex ante mutually beneficial asset trades result from the maturity structure per se. However, ex post mutually beneficial exchanges arise from the different rates of time preference. These trades involve impatient agents exchanging assets for consumption goods. We derive conditions such that trades are more likely to take place if the assets held by impatient agents are more easily verifiable below. As a consequence, impatient agents place a higher value on the verifiability of assets which in turn opens up the possibility for mutually beneficial trades also in the initial trading period.

Each period, agents are matched randomly in pairs. For each match, each agent decides whether he or she wishes to trade. A trade occurs if both agents accept the (pre-determined) terms of trade. Consider the potential

⁶ The indivisibility assumption has been used by the literature to eliminate the bilateral bargaining problem. See, e.g., [9, 16]. A recent paper [14] focuses on the bargaining problem.

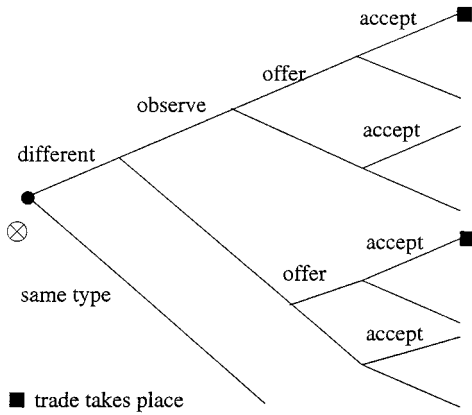


FIGURE 1

trades. In the first period, no trees have yet borne fruit and therefore only asset trades are possible; i.e., an asset can only be traded for another asset.

The decision tree in Fig. 1 illustrates market activity in the first period for a patient agent. (The decision tree is identical for an impatient agent.) The lower node represents the event that he meets another patient agent, in which case no mutually beneficial trades are possible. The upper node is the event that he meets an impatient agent, who may or may not observe the asset held by the impatient agent. In either case, the patient agent decides whether or not to offer his asset for trade. Only when the impatient agent accepts the offer does a trade actually take place. All other outcomes involve each agent holding on to their original asset.⁷

In the second period, some assets will have paid off. Consequently, matches are two agents carrying assets, two agents carrying consumption goods, or one agent carrying the consumption good and the other one an asset. Goods are assumed to be indivisible. The decision tree in Fig. 2 describes the alternatives.

If the asset of a patient agent (argument is parallel for an impatient) has paid off, we are on the lower node. If the patient agent received a large fruit (L), he consumes it. In the event that the fruit is small (S), the patient agent might want to trade. If he meets someone with an asset, the decision tree is identical to that of the first period asset market starting at \otimes . If the other party carries a fruit, there is no inference problem, so trades take place if agents are of different types and both are willing to trade. The upper node represents the event that the asset did not pay off in period two. The patient agent can either meet someone carrying an asset, the

⁷ Since trades require the voluntary participation of both agents and the terms of trade are fixed, it makes no difference whether agents choose to trade simultaneously or sequentially.

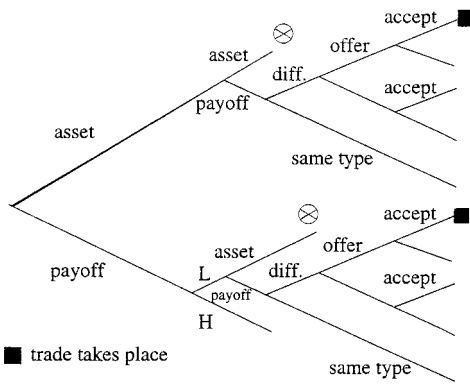


FIGURE 2

upper branch, or someone carrying a fruit, the lower branch. In the former case, the structure from first-period trading is repeated starting at \otimes . In case the other party carries a fruit, the verification is automatic and the tree otherwise follows the same structure as before. In the third period, agents who left the second period with assets simply consume the fruit and no trades take place.

Each agent chooses a trading strategy to maximize the expected discounted utility from consumption, taking the strategies of other agents as well as the distribution of assets in the population, \mathbf{d} , as given. The trading strategy of an agent will be a function of the asset that agent is carrying, $i = (q, n)$ and the perceived asset of the agent with whom he is matched, $j = (q', n')$. Note that this allows situations where the agent decides to offer an asset for trade, although the asset of the other party is not observed in which case asset j is a distribution of quality of assets conditional on n' . Let $\tau'_k(i, j) = 1$ if an agent of type k in period t wants to trade what he is carrying, i , for what the counterparty is perceived to be carrying, j , and zero otherwise. It is clear that agents of the same type will never trade, $\tau'_k \tau'_k \equiv 0$, and a double coincidence of wants is needed for a trade to take place, $\tau'_k(i, j) \tau'_i(j, i) = 1$. The equilibrium is defined as follows.

DEFINITION 1. An equilibrium is a set of trading strategies $\tau'_k \rightarrow \{0, 1\}$, one for each type of agent $k \in \{P, I\}$, together with distributions of assets $d = \text{dist}\{(q'_P, n'_P), (q'_I, n'_I)\}$ that satisfy

- (i) maximization, so that each agent chooses trading strategies that maximize expected utility given the trading strategies chosen by other agents and given a second-period distribution of assets, d ,

- (ii) rational expectations, so that the strategies chosen, $\{\tau_k^t\}$, produce the second-period distribution of assets, d ,
- (iii) Bayesian updating, so that agents revise their priors on the distribution of assets d taking any new information into account.

3. EQUILIBRIUM TRADING STRATEGIES

There are two types of possible equilibria in the model. The first one is an equilibrium in which trade only takes place when and if both parties have full information. In the second kind of equilibrium, trades take place in spite of an asymmetry of information between the buyers and sellers of assets. We consider each in turn.

3.1. *Informed Trading*

We will derive equilibrium trading strategies by considering potential trades in each period, starting from period two and moving backwards. Agents whose asset has given a large payoff in the second period will clearly not want to trade their food for any asset. Time discounting will make them better off by simply consuming the crop and leaving the market. Trade will not take place when two agents of the same type meet, regardless of whether they both have fruit or assets, or if one carries an asset and the other one carries fruit. A double coincidence of wants is required for trades to take place. If there is no difference between agents' preferences, no mutually beneficial trades are possible. It is also easy to see that no trades will take place between an impatient agent with a small fruit and a patient agent with an asset.

When it comes to pure asset trades between agents of different types, three situations need to be discussed. Everyone knows that all trees still around in the second period will pay off in the final period. This implies that no agent is willing to pay a premium for verifiability in the context of pure asset trades in the second-period market. It is obvious that if both agents have observed each others' asset, no such trades will take place in the second period. Trades asset for asset are also precluded when only one agent is able to observe the other party's asset. The fully informed agent will only want to trade if the partially informed agent's asset is of higher quality. That is a signal to the partially informed agent that she is being ripped off. If neither agent sees the quality of the other party's asset, trades are precluded by the standard rational expectations argument.

Consequently, the only trades that may occur in the second period are those between a patient agent with a small fruit (S) and an impatient agent with an asset. Suppose that the quality of the asset of the impatient agent

is observed. If the quality of the asset of the impatient agent is sufficiently low, it might be better for the patient agent to consume the small fruit. Similarly, if the quality of the asset is sufficiently high, it might be better for the impatient agent to hold on to the asset.

DEFINITION 2. The second-period asset market trading set, conditional upon asset's quality being verified, is defined as

$$T \equiv \left\{ q \mid \left(\underline{q} \equiv \frac{S}{L-S} \frac{(1-\beta_P)}{\beta_P} \leq q \leq \min \left(1, \frac{S}{(L-S)} \frac{(1-\beta_I)}{\beta_I} \right) \equiv \bar{q} \right) \right\}.$$

If an impatient agent with an asset with $q \in T$ meets a patient agent with S and the asset is observed, a trade would take place.

Should the quality of the impatient agent's asset not be observed, however, the patient agent makes his trading strategy contingent on the expected quality of assets among impatient agents, conditional on trade being suggested. Since first-period trading might change the distribution of assets across the two groups of agents, we define the distributions of assets after initial asset market trading for patient agents as $G_P(q, n)$ and for impatient agents as $G_I(q, n)$. It will be shown later that $\int_0^1 q dG_I(q, n) \leq \int_0^1 q dF(q, n)$. A sufficient condition to preclude trade without verification is that the expected quality in the initial distribution conditional on any n is lower than \underline{q} . We shall assume this to be the case.

Assumption 2. $\int_0^1 q dF(q, n) \leq \underline{q}$ for all n .

Prior to matching in the second period, the expected value to a patient agent of his asset depends on the likelihood that a successful trade takes place. The expected value, $W_P(q)$, of an asset of quality $q \in T$ carried by a patient agent is then

$$W_P(q) = (\delta + (1-\delta) \beta_P) \pi(q) + \mu_I(1-\delta) \delta(1-q) \int_{q' \in T} n' [\beta_P \pi(q') - S] dG_I(q', n'), \quad (3.1)$$

where (q', n') represents an impatient agent's asset.

The first part is the expected value of an asset that does not get traded. The second part is the expected value of the option to trade the asset in period two. The probability of meeting an impatient agent with whom a successful trade can take place is $\mu_I(1-\delta)$. Trading only takes place when the asset pays off a low amount in the second period, which happens with probability $\delta(1-q)$. The integral takes into account the fact that a trade only takes place if the asset in the hands of the impatient agent is of sufficient quality (that is, $q' \in T$) and is verified. Such a trade gives the patient

agent an asset with the discounted value $\beta_P \pi(q')$ in exchange for the small payoff S .

We will now derive an expression for the expected value of an asset (q, n) to an impatient agent. Let $W_I(q, n)$ denote the expected value of an asset with quality $q \in T$ and verifiability n in the hands of an impatient agent,

$$W_I(q, n) = (\delta + (1 - \delta) \beta_I) \pi(q) + \mu_P \delta \left(1 - \int_0^1 q' dG_P(q', n') \right) n(1 - \delta) [S - \beta_I \pi(q)]. \quad (3.2)$$

The first part reflects the expected value of an asset that does not get traded. The second part represents the option value of second-period trading. Such a trade will only take place if the asset did not pay off in the second period, which happens with probability $(1 - \delta)$. For convenience of notation, let $\alpha \equiv \mu_P \delta (1 - \int_0^1 q dG_P(q, n))$ be the probability that an impatient agent meets a patient agent with a small fruit in the second period. Note that the expected quality among patient agents in the second period depends on first-period trading and consequently α will have to be determined in equilibrium. The probability of a successful trade is $\alpha n(1 - \delta)$ and the trade gives the impatient agent S in return for the expected discounted value of the asset $\beta_I \pi(q)$. Note that $\partial^2 W_I(q, n) / \partial n \partial q < 0$, so the marginal contribution of verifiability to the expected value of an asset is lower for higher quality assets.

Now step back one period to study pure asset trades in the first period. Recall that the initial distribution of quality and verifiability of assets is independent of type. Two agents of the same type will thus not trade, regardless of whether the quality of their assets are observed. Only if two agents value the same assets differently are there gains to be made from trade. Since impatient agents will only be able to exchange an asset for a payoff in the second period if that asset can be observed by the patient agent, impatient agents will generally care about the verifiability of an asset. They might accept a lower quality tree in the initial asset market provided that it has a higher level of verifiability. This difference in valuation forms the basis for the existence of an initial asset market where assets are traded for other assets.

Trades thus take place exclusively between patient and impatient agents also in the initial asset market. The willingness of impatient agents to give up quality for verifiability clearly depends on the probability that they will be able to execute a successful trade in the second period. Patient agents do not value verifiability of purchased assets in first-period trading since they will only trade in the second period if their asset has paid off.

Our assumptions imply that there is no uninformed trading in the second period. We will now develop conditions such that no uninformed

trading takes place in first period either. From Assumption 2, it follows that a patient agent will never offer to exchange an asset of quality higher than \underline{q} in first-period trading unless they observe the quality of the impatient agent's asset. The impatient agent might, however, be willing to give up an asset of quality above \underline{q} in first-period trading even though she cannot observe the quality of the patient agent's asset. This would occur if the expected benefit from improving the verifiability is sufficiently large to offset the expected loss in quality.

Recall that the verifiability of assets is observable. Let B_1 be the expected benefit from trade to an uninformed impatient agent conditional on the verifiability of the asset proposed for trade, n' ,

$$B_1(q, n; n') = \int_0^q [W_1(q', n') - W_1(q, n)] f(dq' | n'), \tag{3.3}$$

where f is the marginal probability distributions of q' conditional on n' . Consider an impatient agent with a non-verifiable asset, $n = 0$. The best this agent can do is to obtain an asset that is fully verifiable, $n' = 1$. The expected benefit would be

$$B_1(q, 0; 1) = \int_0^q W_1(q', 1) f(dq' | 1) - W_1(q, 0). \tag{3.4}$$

To maximize the benefits from verifiability, set $\alpha = 1$. Note that an asset of quality below \underline{q} has zero second-period option value. The above expression can then be written as

$$B_1(q, 0; 1) \leq \int_0^q (\delta + (1 - \delta) \beta_1)(\pi(q') - \pi(q)) f(dq' | 1) + \int_{\underline{q}}^q (1 - \delta)(S - \beta_1 \pi(q')) f(dq' | 1). \tag{3.5}$$

A sufficient condition to prevent the impatient agent from trading an asset $q > \underline{q}$ without information is that $B_1(q, 0; 1) \leq 0$. We shall assume this to be the case.

Assumption 3. $\int_0^q [(\delta + (1 - \delta) \beta_1)(\pi(q') - \pi(q)) f(dq' | 1) + \int_{\underline{q}}^q (1 - \delta)(S - \beta_1 \pi(q')) f(dq' | 1)] \leq 0$.

Thus, the only trades that can take place without full information is when at least one uninformed agent has an asset below \underline{q} . Since preferences coincide in this set, no such trades will take place.

PROPOSITION 1. *Under Assumptions 2 and 3, no trades take place unless both agents observe the quality of each others' asset.*

We emphasize that all trades in the initial market require the quality of both assets to be observed. The value of an asset in the initial market depends on the likelihood of a successful trade in the following period. The set of assets that are eligible for trade in initial market is thus restricted to $q \in T$. If the impatient agent carries a tree with quality $q_I \geq \bar{q}$, she will not be willing to give up quality for verifiability since she optimally holds on to such an asset in period one. If, on the other hand, $q_I \leq \bar{q}$, no patient agent will take the tree for fruit in period one and verifiability is not an issue. It is only for the assets in the set T that agents value the characteristic of the assets differently and can thus engage in mutually beneficial trades.

For trades to be mutually beneficial, the patient agent must get a tree of higher quality and the impatient agent must be better off with the acquired combination of quality and verifiability than she is with her old asset.

DEFINITION 3. The set of potential trades between a patient agent with (q_P, n_P) and an impatient agent with (q_I, n_I) in initial period asset markets is defined as

$$C(\alpha) \equiv \left\{ (q_I, n_I, q_P, n_P) \left| \begin{array}{l} \text{(i) } q_I \geq q_P \\ \text{(ii) } W_I(q_P, n_P) \geq W_I(q_I, n_I) \\ \text{(iii) } q_P \in T, \end{array} \right. \right.$$

where the dependence of W_I , on α is understood.

Equation (3.2) can be used to define the indifference curves of an impatient agent in (q, n) -space. The indifference curve of the impatient agent is negatively sloped and concave below \bar{q} . As mentioned above, the patient

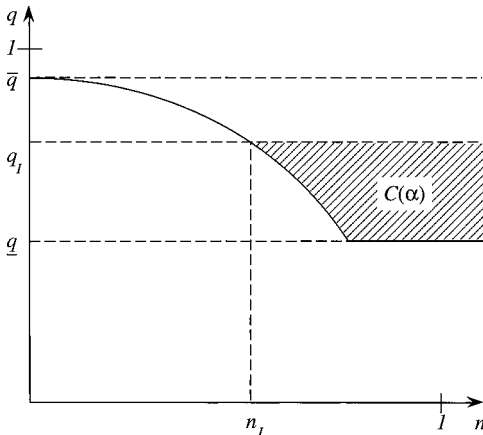


FIGURE 3

agent only cares about the quality of the asset which makes his indifference curves horizontal. Figure 3 illustrates a section of the set $C(\alpha)$ for an impatient agent with (q_1, n_1) .

3.1.1 Equilibrium with Informed Trading

With first-period asset trades taking place, the second-period distributions of quality and verifiability of assets across types of agents are endogenously determined. Decision rules of agents in the first period depend on the likelihood of meeting a person with whom a trade can take place in the second period. This likelihood is a function of the endogenously determined second-period distribution of quality of assets, which in turn depends on the trading rules for initial period trading. Proposition 2 ensures that an equilibrium with liquidity trading exists.

PROPOSITION 2. *Under Assumptions 2 and 3, there exists a unique equilibrium with liquidity trading.*

Proof. Consider a patient agent with asset (q, n) , $q \in T$, at the beginning of the first trading period. His end of period asset, (q', n') , will be a random variable with joint distribution that depends on trading strategies. For given $\alpha > 0$, let $D(q, n; \alpha) = \{(q', n') | (q', n', q, n) \in C(\alpha)\}$, where the dependence of C on α follows from Definition 3. This gives the set of possible end of period asset holdings for this agent, should asset trading take place. The larger $D(q, n; \alpha)$ is, the more matches will result in trades. Since each trade results in $q' \geq q$, this increases the average end of period q' . Denote this conditional expectation $E(q' | q, n; \alpha)$.

Figure 4 plots the set $D(q, n; \alpha)$, which is given by the intersection of the lower countour set of (q, n) for an impatient agent with the set $\{q' \geq q\}$. It

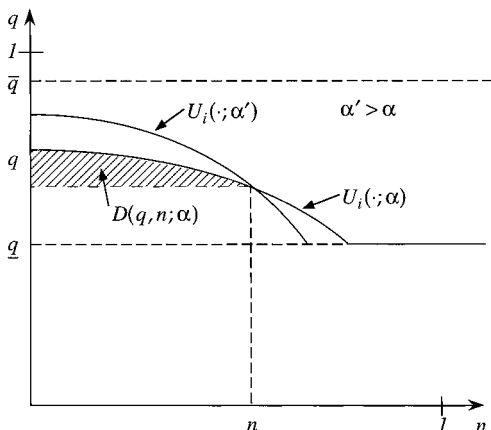


FIGURE 4

is straightforward to verify from Definition 3 that as α increases, the indifference curves for an impatient agent rotate as shown in Fig. 4, thus enlarging the trading set. So, $E(q' | q, n; \alpha)$ is nondecreasing in α and, as shown in Lemma 1 (Appendix), it is also continuous. As a result, \hat{q} , the second-period average q for patient agents is also continuous and nondecreasing in α . Denote this function $\hat{q} = Q(\alpha)$. The probability of a successful match depends on the proportions of agents in the market being eligible for trade: $\alpha = \mu_P \delta(1 - \hat{q})$ defines a decreasing mapping $P(\alpha) = \mu_P \delta(1 - Q(\alpha))$ on $[0, 1]$ whose fixed points determine equilibria. Since P is continuous and decreasing, there exists a unique equilibrium point. ■

Note that trading in the initial stock market takes place because people are forward looking. They take future trading opportunities into account when assessing the value of an asset. The only heterogeneity across agents is their rate of time preference. This causes agents to evaluate characteristics of assets differently. An impatient agent is more concerned about being able to liquidate an asset early on and thus puts a higher price on the feature of assets that make them more liquid—verifiability. The outcome of the initial stock market is a redistribution of assets among the groups of patient and impatient agents. Specifically, the average verifiability of assets among impatient agents improves while the average quality of their assets falls. Note that this implies that $\int_0^1 q dG_I(q, n) \leq \int_0^1 q dF(q, n)$, which together with Assumption 3 justifies the conjecture that no uninformed trading takes place in the second period. For patient agents, on the other hand, the average quality of assets improves through trading while the average verifiability of assets fall.

3.1.2. Return Dominance with Informed Trading

To derive a notion of asset return it is convenient to have an expression for the expected utility of patient and impatient agents with an asset of characteristics (q, n) , denoted $U_P(q, n)$ and $U_I(q, n)$ respectively, measured in terms of second-period consumption.

$$U_P(q, n) = W_P(q) + \mu_I n \int_{\{(q', n', q, n) \in C\}} n' [W_P(q') - W_P(q)] dF(q', n'), \quad (3.6)$$

$$U_I(q, n) = W_I(q, n) + \mu_P n \int_{\{(q, n, q', n') \in C\}} n' [W_I(q', n') - W_I(q, n)] dF(q', n'). \quad (3.7)$$

In Eqs. (3.6) and (3.7), the first term measures the conjectured value if no trades take place in the first-period asset market while the second term measures the value to each agent from having access to the stock market in the first period.

The standard notion of the expected return to an asset is not defined in an environment where there are no market prices. We can, however, measure the expected shadow return for an asset using the indirect utility function of agents. This shadow return depends on who is initially endowed with the asset in question. In general, the expected shadow return $(1 + r_k)$ for an agent of type $k \in \{P, I\}$ will be defined as the internal rate of return that equates the expected payoffs to an asset with the indirect utility from holding the asset,

$$U_k(q, n) = \pi(q) [\delta + (1 - \delta)/(1 + r_k)], \quad (3.8)$$

where the left-hand sides are Eqs. (3.6) and (3.7) for a patient and an impatient agent respectively. It follows that the shadow rate of return is increasing in $(U_k(q, n)/\pi_q)^{-1}$. Since $\pi(q)$ is independent of n and $U_k(q, n)$ is increasing in n for $q \in T$, the shadow rate of return is decreasing in n regardless of the type of its owner.

With no first-period trading, it is easy to verify that the shadow returns for both the patient and the impatient agent will be increasing in the quality of the asset.

An asset that is not tradable has an expected shadow return of $(1 + r_k) = 1/\beta_k$. An asset in the trading set, $q \in T$, will have a lower expected shadow return (or a higher shadow price) because of the option value of trading in the future. Since the value of an asset to an agent depends on the opportunities for exchanging the asset in the future, the shadow return also depends on the observability of the asset. We summarize the properties of the expected shadow return in Proposition 3.

PROPOSITION 3. *The expected shadow rate of return on a tradable asset, $q \in T$, is always decreasing in n . With no first-period trading, the rate of return is also increasing in q irrespective of the type of its initial owner.*

Tradable assets have a liquidity premium due to the option value of future potential trades. This liquidity premium is decreasing in the quality of the asset. Since successful trades require full information for both the buyer and the seller, an asset whose quality is more easily observed is more valuable. Such an asset has a higher liquidity premium, and correspondingly a lower expected shadow rate of return.

The equilibrium also has implications for the relationship between liquidity premia and trading volume. Even without considering first-period trading, assets with higher n will on average trade more in the second period than those with lower n . Moreover, since assets with higher n are needed as "store of liquidity" they are more likely to be traded in the first period. Assets with a liquidity premium are thus also assets that will tend to be traded more frequently in equilibrium.

3.1.3. Discussion

The literature on rational expectations has also emphasized the cost to liquidity traders that results from information asymmetries. This cost arises since uninformed traders (or intermediaries) need to be compensated for the losses that may arise from the opportunistic behavior of those that are informed. The consequence is larger bid–ask spreads for assets where information asymmetries are more pervasive. In our model the information asymmetry affects instead the ability of executing trades.

There are two characteristics of our model which may account for this difference. One is that the set of possible trades is limited by the indivisibilities in the exchange. The other one is that our impatient agents are liquidity traders only in the extreme case when $\beta_I = 0$. As long as agents are not extremely impatient, there will generally be a tradeoff between liquidating assets at a low price and waiting for better future opportunities of exchange.

Though the indivisibility accounts for some of the unexecuted trades, we will show that information asymmetries can result in consumption of liquidity traders being delayed even when agents are able to make side payments. For this purpose, we will only consider second-period trades. Consider a situation in which a patient agent with small fruit meets an impatient agent with an asset and is not able to verify its quality. Let \hat{q} denote the expectation of the quality of the impatient agent's asset conditional on her willingness to trade. From Assumption 2 it follows that $\hat{q} < q$, so the patient agent is not willing to trade. Letting τ denote the smallest transfer⁸ that the impatient agent needs to make so that trade takes place, it is easy to show that

$$\tau = (q - \hat{q})(L - S). \quad (3.9)$$

Since the gains to trade for the impatient agent are decreasing in q , for a given value of τ only those with q below some value $Q(\tau)$, where $Q(\tau)$ is implicitly defined by

$$S - \tau = \beta_I(S + Q(\tau)(L - S)), \quad (3.10)$$

will be willing to make such side payment and trade. It follows that $Q(0) = \bar{q}$. Finally, in equilibrium

$$\hat{q} = E_{q \leq Q(\tau)}(q). \quad (3.11)$$

⁸ These can be thought of transfers of a perfectly divisible endowment that enters linearly in the utility function.

Under assumption 2 it is easy to show that there exists an interior equilibrium⁹ and that all equilibria involve strictly positive transfers. In addition, if $\bar{q} < 1$ then $Q(\tau) < \bar{q}$. The latter implies that some trades that would take place under perfect information would not take place if the match is with an uninformed agent. Thus, allowing for transfers reduces the probability of no trade arising from the asymmetry of information but in general does not reduce it to zero. Note however that in the special case where $\beta_1 = 0$, in equilibrium $Q(\tau) = 1$ and consequently all trades take place. This corresponds to the extreme case of liquidity trading generally modeled in the literature. If impatient agents are not *too impatient*, some trades will not take place.

It is interesting to note that our result on return dominance relies on the fact that the asymmetry of information can result in no trade. This happens because the potential inability to trade introduces an ex ante subjective cost which is not captured in the ex post measured terms of trade. Consequently conventional models of liquidity trading do not exhibit the return dominance of liquid assets which we derive.

3.2. Uninformed Trading

Our previous analysis established that when trading only takes place between informed agents, verifiability of an asset enhances its tradeability and thus its shadow price. This in turn implies that the expected rate of return on less verifiable assets is higher to compensate for the lack of tradeability. In this section we study the case where second-period transactions occur in absence of asset quality verification.

Let \hat{q}_1 be the expected value of q for impatient agents trading assets in the second period. A patient agent will trade in absence of verification provided that $\hat{q}_1 > \underline{q}$. Since \hat{q}_1 not only depends on the initial distribution but also on first-period trades, it is not enough to assume this property for the unconditional mean. However it is simple to impose restrictions on the distribution of agents types and on preferences to derive this result. As an example, if $\beta_P = 1$, and $\underline{q} = 0$ the condition immediately follows.

PROPOSITION 4. *If $\hat{q}_1 > \underline{q}$ there exists an equilibrium with uninformed trading.*

Proof. See Appendix.

What trades will occur in the first period? All assets with $q > \underline{q}$ are fully tradeable in the second period, while assets with $q < \underline{q}$ will be traded only if they are not verified. Consequently, only impatient agents with $q \in [0, \underline{q}]$

⁹ Stronger assumptions are needed on the distribution of q (e.g., concave distribution functions) for the equilibrium to be unique.

will trade in the first period. The purpose of these trades will be to *lower verifiability*, thus increasing the probability of exchange in the following period. Incidentally, note that since the maximum reduction in q that could take place in the first period is \bar{q} , the assumption in the above proposition will be satisfied if, for instance, $\bar{E}[q|n] > 2\bar{q}$. Will uninformed trading take place in the first period? Again, it is easy to find conditions under which this will always occur. For example, if the initial distribution of assets is uniform on $[0, 1] \times [0, 1]$, such trades will take place.

For assets in $[\bar{q}, 1]$, verifiability has no effect on trading and thus the rate of return is independent of n . In contrast, for assets in $[0, \bar{q})$ higher n will make the asset *less tradeable*. The shadow price for these assets will be decreasing in n and consequently their rate of return increasing. Agents with low quality assets will only be able to trade if their assets remain undetected. When uninformed trading takes place, it is thus the “lemons” or low quality assets which are less known in the market that carry a liquidity premium. Liquidity trading in the first period will again imply that assets which have a liquidity premium will be traded more frequently.

3.3. Uniqueness

We have seen that under Assumptions 2 and 3 there is a unique equilibrium and that this equilibrium involves informed trading only. The example given below shows that the uniqueness result does not hold in general. The basic intuition behind the example is the following: Consider a situation where the distribution of asset holdings of impatient agents is such that uninformed trading would take place in the second period. If impatient agents anticipate this, they may not have much incentives to trade in the first period. However, suppose that impatient agents expect that uninformed trades will not take place in period 2. They then have a motive for exchange in period 1. This exchange lowers the average quality of the assets held by impatient agents. If it lowers it sufficiently, then no uninformed trading will take place in the second period, thus fulfilling the impatient agents’ expectations.

EXAMPLE 1. Let there be three assets $\{(q_1, n_1), (q_2, n_2), (q_3, n_3)\}$ with the following properties:

- (i) $\bar{q} > q_1 > q_2 > \underline{q} > q_3 = 0$,
- (ii) $n_1 = \frac{1}{2}$, $n_2 = 1$, and $n_3 = 0$,
- (iii) $\iota_1 = \iota_3 = \frac{1}{2}$, and $\iota_2 = 0$, $p_2 = 1$, and $p_1 = p_3 = 0$,

where ι_i denotes the initial distribution of impatient agents asset holdings and p_i that corresponding for patient agents.

Case 1. No trading in the first period. Consider second-period trading conditional on no observation. The posterior distribution for the asset held by the impatient agent is $\{P(q_1) = \frac{1}{3}, P(q_3) = \frac{2}{3}\}$. The following assumption ensures that trading takes place:

$$(iv) \quad \frac{1}{3} q_1 + \frac{2}{3} q_3 > \underline{q}.$$

Trades occur with probability $\frac{1}{4} \delta(1 - \delta)(\frac{3}{4})$ and generates an average net welfare gain of $(\frac{1}{3} \pi(q_1) + \frac{2}{3} \pi(q_3))(\beta_P - \beta_I)$. It is interesting to note that if patient agents are willing to exchange without verifying in the second period, no first-period trades can benefit impatient agents, and thus this is an equilibrium for the matching game.

Case 2. Trading in the first period. We will construct another equilibrium where, as a result of first-period trading, the distribution of second-period asset holdings of impatient agents is downgraded enough to kill uninformed trading. The equilibrium will involve only informed trading in the second period, provided $q_1 - q_2$ is not too large which we shall assume to be the case. Since $n_2 > n_1$, impatient agents will now be willing to trade in the first period.

Let τ denote the fraction of impatient agents holding assets of type q_1 that end up trading in the first period. Consider second-period trading conditional on no observation. The corresponding distribution for the asset held by the impatient agent is now $\{P(q_3) = 2/(3 - \tau), P(q_1) = (1 - \tau)/(3 - \tau)\}$. With the following assumption, no uninformed second-period trading will take place:

$$(v) \quad (1 - \tau)/(3 - \tau) q_1 + 2/(3 - \tau) q_3 < \underline{q}.$$

Note that this assumption is not inconsistent with all of the above and that this is also an equilibrium for the matching game. Incidentally, note that the absence of uninformed trading in the second period results in a welfare loss, which is partly offset by the trades that take place in the first. These gains will not be enough to offset the losses, provided $q_1 - q_2$ is not too large. In that case, the equilibrium described in Case 1 gives higher welfare.

4. WELFARE

Second-period trading unambiguously improves welfare in this economic environment. To see this, assume that $\mu_P = \mu_I = \mu$. The total social gain from second-period trading is given by $\mu \delta(1 - \delta)(1 - \int q dF(q, n)) \int_{q' \in T} n'(\beta_P - \beta_I) \pi(q') dF(q', n') \geq 0$. First-period trading does not generate welfare gains in itself, since all assets have ex ante the same maturity structure. However, these trades can affect the likelihood of second-period trading and thus have welfare effects. These effects can be negative, as the

above example has indicated. This example relied on the existence of an equilibrium with uninformed trading. Even when such equilibria do not exist, it cannot be shown that first-period trading is pareto improving. Though all trades are individually rational, agents are atomistic and thus do not take into account the effect that their individual first-period trades have on overall trading opportunities in the future. In the working paper version of the paper, we provide an example where impatient agents' expected utility is lowered by the availability of first-period trading.

For patient agents, the externality caused by first-period trading is not unambiguously negative. First-period asset trading improves the likelihood of a successful trade in the second-period market. At the same time, the average quality of assets that patient agents can receive in return for S is poorer due to first-period trading. Note that an asset can change hands at most twice in this world. A relatively poor quality asset that was exchanged for a better quality asset in first-period trading can make its way back to the patient agents via second-period trading. On net, the trades should still be beneficial for the patient group as a whole. The better quality asset received in the first market is expected to give a higher crop and it might do so already in the second period. The poor quality asset will return to the patient group only in exchange for a low payoff and it will affect only third-period consumption. Due to discounting, the patient agents gain from asset market trading. We conjecture that patient agents as a group never lose from having access to an asset market when all equilibria involve informed trading.¹⁰

For the remainder of this paper, we will focus on the case when only informed trading takes place. Thus, more information is a good and there are no incentives to hide the quality of assets.

5. IMPROVING THE TRADING ENVIRONMENT

In the equilibrium of the random matching game there are many mutually beneficial trades which are left unexecuted. This is partly due to the information asymmetry and to the lack of coordination which underlies random matching. There are obviously incentives for at least some agents to reduce the information asymmetry by seeking to increase the market's awareness of their assets. This section explores one alternative costly mechanism for achieving that goal. There are also gains that may arise from coordinating trades which need to be balanced against the costs of intermediation. This section also explores equilibria where agents at a cost

¹⁰ This is not true when equilibria involves uninformed trading, as shown by an example included in the working paper version of this paper.

are offered to participate in alternative more centralized mechanisms of intermediated trading.

We endow each agent with a perishable perfectly divisible consumption good in the initial period. This good can be used to pay participation fees. To make things simple, assume that this good enters linearly into the agents' preferences. Given costly participation, not all agents will choose to use the mechanisms available for reducing trading frictions. Thus, a market will generally co-exist with intermediation. To simplify the analysis, first-period asset market trading will be ruled out. This does not change the qualitative nature of our results. Since second-period trading is unambiguously welfare improving, this allows us to focus on the role of intermediation in reducing the frictions associated with the trading environment.

5.1. *Reducing the Information Problem*

Consider introducing a technology whereby the verifiability of an asset can be improved at a variable cost, c , per unit of increase in n . Since increased verifiability improves future trading opportunities for tradable assets, agents with potentially tradable assets would be willing to pay something at the outset for being able to increase the verifiability of their specific asset. This investment can be thought of as improving accounting techniques, implementing disclosure practices which increase the transparency of the asset to a potential buyer, or buying services from an investment bank which gives credence to the asset. It seems reasonable to assume that costs associated with such efforts are higher for securities that are initially less well known.

Which agents will invest more in verifiability? Since most of the benefits associated with higher n derive from second-period trading, let us first concentrate on these trades. The corresponding option value for a tree (q, n) to an impatient agent is given by $\alpha n(1 - \delta)(S - \beta_1 \pi(q))$, which is linear in n . The marginal value of n is $\alpha(1 - \delta)(S - \beta_1 \pi(q))$ which is decreasing in q . Agents with higher quality trees are less likely to want to trade in the second period since the probability of them ending up with a small fruit is lower. As a result, impatient agents with higher quality trees have less of an incentive to invest in verifiability.

Higher verifiability increases the likelihood of a trade taking place upon a match. Since the surplus obtained by this transaction will generally be shared between the two agents, the private incentives for investment will generally not produce the socially optimum level. This is a standard issue in search models. For our particular model, one may expect investment to be too small. Again this can be easily seen by focusing on the benefits derived from second-period trading. The marginal gains from investment

for an impatient agent are $\alpha(1-\delta)(S-\beta_1\pi(q))$ while the social marginal gain is $\alpha(1-\delta)(\beta_p-\beta_1)\pi(q)$. Since for a trade $q > \underline{q}$, $\beta_p\pi(q) > S$, so $(\beta_p-\beta_1)\pi(q) > S-\beta_1\pi(q)$. At an interior equilibrium, investment would fall short of the optimum. This could rationalize minimal disclosure requirements similar to those stipulated for listed securities by the SEC.

5.2. Intermediated Trading

We now discuss the possibility of some alternative and more centralized mechanisms for exchange. Our purpose is to capture some important features of intermediation in order to highlight how reducing the frictions in the previously described trading environment might impact on trading opportunities and the welfare of agents. We study two centralized mechanisms. The first allows agents to choose the timing for their consumption. The second enables impatient agents to borrow against future income in the second period. Agents can choose to remain outside the intermediated structure if they find the participation costs too onerous. We assume that agents wishing to participate must pay a fee to the intermediary to cover its costs for verifying the deposited asset. We will assume free entry so that the fee charged for verifying assets in each case is equal to the verifying cost.

5.2.1. *Ex Ante Mechanism for Intertemporal Reallocation*

Suppose that agents can take their endowed assets to a firm which at a fixed cost c gives in exchange a future payoff at a fixed date: for impatient agents in the second period and for patient agents in the third period. The intermediary serves several functions: it eliminates trading frictions in second-period markets by providing central clearing, it eliminates the costs associated with information asymmetries, and it pools the risk of the maturity structure of assets. The services provided by the intermediary resemble those typically provided by mutual funds.

Consider an intermediated contract that specifies an interest rate $1/\beta$ where $\beta_I < \beta < \beta_P$ such that an agent with an asset of quality q can choose to consume a lottery with expected value $(\delta + (1-\delta)\beta)\pi(q)$ in the second period or one with expected value $(\delta + (1-\delta)\beta)\pi(q)/\beta$ in the third period. Note that the expected payoffs to the lottery are conditional on the quality of the deposited asset, q . Obviously impatient agents will choose to consume in the second period and patient agents in the third. We now establish how the interest rate is determined.

Assume first that the set of agents (q, n) that participate in this arrangement are A_P for patient agents and A_I for impatient. Let $\pi_P = \mu_P \int_{A_P} \pi(q) dF(q, n)$ and $\pi_I = \mu_I \int_{A_I} \pi(q) dF(q, n)$. The resource constraint for the mutual fund is given by $(\delta + (1-\delta)\beta)\pi_I = \delta(\pi_I + \pi_P)$ where

the left-hand side represents the demand for second-period payoffs and the right-hand side the supply.¹¹ From this constraint we obtain the expression

$$\beta = \frac{\delta}{1 - \delta} \frac{\pi_P}{\pi_I}. \quad (5.1)$$

Note that in spite of the linearity of preferences and the different discount rates, the interest rate is pinned down by the participation of agents. We now turn to the joint determination of β and the participation sets A_P and A_I . The benefits from participation are

$$B_I(q, n) = (\beta - \beta_I)(1 - \delta) \pi(q) - c - \alpha_P(1 - \delta) n(S - \beta_I \pi(q)), \quad (5.2)$$

$$B_P(q) = \frac{(\beta_P - \beta)}{\beta} \delta \pi(q) - c - \mu_I \delta (1 - \delta)(1 - q) \int_{A_I^c} n'(\beta_P \pi(q') - S) dF(q', n'), \quad (5.3)$$

where the second term in each equation reflects the expected opportunity cost of giving up the right to participate in the second-period asset market, and α_P is the probability that an impatient agent meets a patient agent, willing to trade, with a small fruit in the second-period market. From the above definitions it follows that

$$\alpha_P = \mu_P \delta \int_{B_P(q) \leq 0} (1 - q) F(dq, dn) \quad (5.4)$$

and

$$A_I = \{(q, n) \mid B_I(q, n) \geq 0\}. \quad (5.5)$$

An equilibrium is given by a triplet (β, A_I, α_P) that satisfy Eqs. (5.2)–(5.5). In the Appendix we sketch a proof of existence of equilibria.

As before, the benefits of participation are decreasing in n , so it is the less verifiable assets that will be traded through this mechanism. In contrast with the previous case, both B_I and B_P are increasing in q , so the higher quality assets will participate. Consider now an increase in the quality of assets held by impatient agents. The direct effect is an increase in π_I . There is also an effect on π_P which is ambiguous, since though less trades will be available in second-period matching, quality will be higher. Provided π_P does not increase by too much, the net effect will lead to a decrease in β ,

¹¹ A similar constraint can be obtained for third-period consumption. However, only one is needed since they are linearly dependent.

i.e., an increase in the interest rate. Similarly, an increase in q for patient agents is likely to result in a lower interest rate. Finally, an increase in the verifiability of assets held by impatient agents will tend to reduce their participation in the fund and consequently make the interest rate decrease.

5.2.2. *Ex Post Mechanism for Intertemporal Reallocation*

In the *ex ante* mechanism, all participants were charged a fee at the outset for the services of the intermediary. *Ex post*, only a subset of those agents will in fact prefer to use the services provided; the patient agents with a small payoff in the second period and the impatient agents which did not get a harvests the second period. In absence of the *ex ante* mechanism for intertemporal reallocation, a viable intermediated mechanism purely for intertemporal reallocation between the second and third periods can exist. The intermediary could be interpreted either as a commercial bank or a "line of credit."

The contract offered by the intermediary would promise patient agents an expected payoff of S/β in the third period and an impatient agent $\beta\pi(q)$ in the second period. Note that the impatient agents get a fixed claim, conditional on the quality of their asset, while the patient agents participate in a lottery. Since it is only the impatient agents who need the verification services, they are assumed to be those to bear the fixed cost of participation, c . What needs to be determined is the interest rate.

Suppose that all patient agents go to the intermediary. This implies that there can be no other trading in the second period. Note that some impatient agents, specifically those with relatively poor assets, might choose to hold on to their assets if the fixed cost, c , is sufficiently high. The demand for asset, or second-period deposits, would then be $\delta\mu_p(1 - \int q' dF(q', n')) S$. Impatient agents expect to get $\beta_1\pi(q)$ by staying in the market while the intermediary offers them $\beta\pi(q) - c$. Define $q(\beta) \equiv \{q | \beta_1\pi(q) \equiv \beta\pi(q) - c\}$. The supply of assets by impatient agents is increasing in β . It is given by $(1 - \delta)\mu_1 \int_{q(\beta)}^1 \pi(q) dq$, which in equilibrium must equal withdrawals by impatient agents. Pick a $\beta \in (\beta_1, \beta_p)$. If demand for assets exceeds supply, an increase in β , i.e., a reduction in the interest rate, will increase supply. Similarly, if supply exceeds demand an increase in the interest rate will balance the demand for funds in the intermediary.

What would happen if at this interest rate some patient agents choose to go to the market instead? Then the equilibrium must involve an interest rate β such that patient agents are indifferent between obtaining S/β and the expected benefits from being in the market. Let α_1 be the set of impatient agents with potentially tradeable assets in the second period who remain in the market and let α_p be the probability of an impatient agent meeting a patient agent with a small fruit in the second-period market. The

expected benefits of participating in the arrangement for a patient and an impatient agent respectively are

$$B_P(\beta) = S(1 - \beta)/\beta - \mu_1(1 - \delta) \int_{\alpha_1} n'(\beta_P \pi(q') - S) dF(q', n'), \quad (5.6)$$

$$B_I(q, n) = (\beta - \beta_1) \pi(q) - c - \alpha_P(1 - \delta) n(S - \beta_1 \pi(q)). \quad (5.7)$$

The first part of B_P is decreasing in β while the first part of B_I is increasing in that same variable. The second term represents the opportunity cost of not being able to use the second-period market when participating in the intermediary. An increase in β makes participation more attractive to impatient agents. The set α_1 will shrink as a result which makes this cost for patient agents smaller.

A candidate equilibrium is a (α_P, β) such that $B_P = 0$.¹² Since B_I is unambiguously increasing in q , there is a q^* for given (α_P, β) such that impatient agents with $q < q^*$ go to the market while those with $q \geq q^*$ have the intermediary verify their assets and exchange them for second-period consumption. Note that it is the assets with low q and high n that remain in the market. These are the ones for which the cost of verifying assets is most onerous.

The above analysis indicates that when intermediated trading is costly, it is the assets with higher payoffs but less visibility which will be traded through such mechanisms. However, this conclusion relies on the fact that verification costs are independent of verifiability. As far as fixed costs of intermediation—unrelated to the specific characteristics of the asset—are important, this qualitative result will still hold.

6. SUMMARY AND CONCLUSIONS

This paper has studied sequential asset trading in a random matching game. We have focused on the situation that arises when agents are not equally informed about the returns of different assets. The model endogenously determines trading strategies for the agents and, in particular, whether an uninformed agent will be willing to trade. These trading strategies imply that in equilibrium some assets are more tradeable than others and through initial trading become reallocated to agents with higher preference for liquidity. Consequently, these assets are more heavily traded and, since they are preferred by agents, have lower rates of return. Our model is thus consistent with the negative relationship between trading volume and asset returns.

¹² It is fairly straightforward to show the existence of such an equilibrium. The proof is available from the authors upon request.

Information asymmetries are costly for those agents that face liquidity needs. In conventional rational expectations equilibrium models, this cost translates into higher bid-ask spreads. In our model this cost can take the form of unexecuted trades. The difference arises from the fact that, in contrast to conventional models of asset trading, we model liquidity traders as rational agents with a higher degree of impatience. Except for the extreme case in which the corresponding subjective discount rate is infinite, impatient agents face a nontrivial intertemporal choice decision. When the cost of trading with an uninformed trader is too high, the agent is better off by delaying consumption.

It is interesting to note that our result on return dominance of liquid assets relies on the fact that the asymmetry of information can result in no trade. This happens because the lack of trade introduces an *ex ante* subjective cost which is not fully captured in the *ex post* measured terms of trade. Consequently, the conventional models of liquidity trading do not exhibit return dominance of liquid assets as our model does. To the extent that the trading friction that arises from information asymmetries takes the form of delayed trading, our result on return dominance will continue to hold.

Because of the sequential nature of trading, transactions that take place at an earlier stage affect the distribution of asset holdings at later stages and thus the corresponding trading strategies. Yet, it is the trading opportunities in the later stages that determine trading strategies in the early ones. This has some interesting theoretical consequences. First, there can be both, equilibria with only informed trading and equilibria with uninformed trading for the same environment. Furthermore, as shown in an example, the latter can lead to higher welfare. Second, in equilibrium all trades are individually rational and thus *ex ante* beneficial to agents. But since agents are atomistic, they do not take into account the effect that their individual trades have on overall trading opportunities in the future. Consequently, it is possible, as indicated by examples in the paper, that even those agents that obtain the largest surplus from these trades could be better off if they were prohibited.

As in all models of decentralized exchange, when deciding to participate in a trade, agents take into account their private surplus only. Thus the private incentives to invest in making assets more tradeable are not necessarily aligned with the social ones. In our model this shows as underinvestment in information, suggesting the potential benefits of minimum disclosure regulations.

Our model has been useful to study some characteristics of asset trading that arise from the sequential nature of trades with informational and trading frictions. In order to do so, we have relied on an extremely decentralized trading structure. Two particular implications of this modeling strategy deserve consideration. First, the equilibrium leaves many oppor-

tunities of mutually beneficial exchange unexploited. This clearly points to the benefits and need of analyzing more centralized intermediation. We briefly explore this in the paper. Second, the assumed indivisibilities in exchange implicitly set the terms of trade exogenously. Though this simplifies the analysis considerably, it leaves out many interesting issues. We believe that it is important to extend the type of model considered here along these lines.

APPENDIX

Proof of Proposition 2 (Continuity of conditional expectation, \hat{q}'_P). Let

$$E[q' | q, n; \alpha] = \mu_1 n \int_{C(q, n; \alpha)} q' n' F(dn', dq') + \left[1 - \mu_1 n \int_{C(q, n; \alpha)} n' F(dn', dq') \right] q.$$

LEMMA 1. *The conditional expectation $E[q' | q, n; \alpha]$ is continuous in α .*

Proof. From Definition 2, it is easy to verify that for any α , $C(q, n; \alpha)$ is closed and that for any decreasing sequence $\alpha_m \rightarrow \alpha$, $\bigcap_m C(q, n; \alpha_m) = C(q, n; \alpha)$ and for any increasing sequence $\alpha_m \rightarrow \alpha$, $\bigcup_m C(q, n; \alpha_m) \supset \text{int}(C(q, n; \alpha))$. Since $F(q, n)$ is continuous, the boundary of $C(q, n; \alpha)$ has probability zero, so $E[q, n; \alpha]$ is continuous in α for each (q, n) . ■

LEMMA 2. *Let $f(x, z)$ be a uniformly bounded continuous function and $g(x, z)$ another continuous function, where $x \in X \subset \mathfrak{R}^n$ and $y \in Y \subset \mathfrak{R}^m$ and both X and Y are compact. Let μ be a continuous measure on X and assume $\mu(g_z^{-1}(\{0\})) = 0$ for all z . Then $H(z) \equiv \int_{\{x: g(x, z) \geq 0\}} f(x, z) \mu(dx)$ is continuous in z .*

Proof. Suppose that $z_n \rightarrow z$. Let χ_n° be the indicator function of $g(x, z_n) > 0$ and χ_n the indicator of $g(x, z_n) \geq 0$. Define similarly χ^0 and χ . Then

$$\int_{\{g(x, z_n) \geq 0\}} f(x, z_n) \mu(dx) = \int \chi_n(x) f(x, z_n) \mu(dx) = \int \chi_n^\circ(x) f(x, z_n) \mu(dx).$$

But $\liminf \chi_n^\circ(x) f(x, z_n) \geq \chi^0(x) f(x, z)$ and $\limsup \chi_n(x) f(x, z_n) \leq \chi(x) f(x, z)$. Applying Fatou's lemma, $\int_{\{g(x, z_n) \geq 0\}} f(x, z_n) \mu(dx) \rightarrow \int_{\{g(x, z) \geq 0\}} f(x, z) \mu(dx)$. ■

Thus, Proposition 2 is proved.

Proof of Proposition 4. Let

$$\begin{aligned}
 G_I(\pi, n, n'; \alpha) &= \lambda(n, \pi) \int_{\{\pi' \geq \pi\}} W(\pi', n'; \alpha) + (1 - \lambda(n, \pi)) x_1 - W(\pi, n; \alpha) \\
 G_P(\pi, n, n'; \alpha) &= \varphi(n, \pi) (E\pi' | W(\pi', n'; \alpha) \leq W(\pi, n; \alpha)) \\
 &\quad + (1 - \varphi(n, \pi)) x_2 - \pi,
 \end{aligned} \tag{6.1}$$

where

$$\begin{aligned}
 \lambda(n, \pi) &= \frac{n \int_{\pi' < \pi} F(d\pi', dn')}{n \int_{\pi \leq \pi} F(d\pi', dn') + (1 - n) y_1} \\
 \varphi(n, \pi) &= \frac{n \int_{W(\pi', n') < W(\pi, n)} F(d\pi', dn')}{n \int_{W(\pi', n') \leq W(\pi, n)} F(d\pi', dn') + (1 - n) y_2} \\
 x_1 &= \int_{G_P(\pi, n) \geq 0} W(\pi, n) F(d\pi, dn) / y_1 \\
 y_1 &= \int_{G_P(\pi, n) \geq 0} F(d\pi, dn) \\
 x_2 &= \int_{G_I(\pi, n) \geq 0} \pi F(d\pi, dn) / y_2 \\
 y_2 &= \int_{G_I(\pi, n) \geq 0} F(d\pi, dn).
 \end{aligned}$$

$G_I(\pi, n, n'; \alpha)$ gives the expected gain for an impatient agent with asset (π, n) from trading with a patient agent with an asset verifiability n' and unknown quality. The first term on the right-hand side involves the expected value of the asset obtained if the patient is *informed*, while the second term gives the expected value if the patient agents is also *uninformed*. In turn, $G_P(\pi, n, n'; \alpha)$ gives the expected increase in π an uninformed patient agent would get from trading.

Define the trading sets

$$\begin{aligned}
 A_{11} &= \{(q, n, q', n') | W(q, n; \alpha) \geq W(q', n'; \alpha) \text{ and } q' \geq q\} \\
 A_{10} &= \{(q, n, q', n') | W(q, n; \alpha) \geq W(q', n'; \alpha) \text{ and } G_P(\pi, n, \pi', n'; \alpha) \geq 0\} \\
 A_{01} &= \{(q, n, q', n') | G_I(\pi', n', \pi, n; \alpha) \geq 0 \text{ and } q' \geq q\} \\
 A_{00} &= \{(q, n, q', n') | G_I(\pi', n', \pi, n; \alpha) \geq 0 \text{ and } G_P(\pi, n, \pi', n'; \alpha) \geq 0\}.
 \end{aligned}$$

A_{11} is the asset of trades between informed agents, A_{10} those between an uninformed impatient and uninformed patient, A_{01} between an uninformed

impatient and informed patient, and A_{00} between both uninformed. The dependence of these sets on α should be understood.

Since the points at which $G_I = 0$ and $G_P = 0$ are generically regular, $P(G_I(\pi, n, n'; \alpha) = 0) = 0$ and $P(G_P(\pi, n, n'; \alpha) = 0) = 0$. So by applying the above lemma repeatedly one can establish that G_I and G_P are continuous in α . We now account for the endogeneity of α .

Let μ denote the product measure of initial asset holdings $F \times F$. Consider first $\bar{q}_P = E_q + \Delta q$, where Δq represents the increase in patient agent's q resulting from first-period trading. The latter is given by

$$\begin{aligned} \Delta q(\alpha) = & \int_{A_{11}} n' n (q' - q) d\mu(q', n', q, n) \\ & + \int_{A_{10}} (1 - n') n (q' - q) d\mu(q', n', q, n) \\ & + \int_{A_{01}} n' (1 - n) (q' - q) d\mu(q', n', q, n) \\ & + \int_{A_{00}} (1 - n') (1 - n) (q' - q) d\mu(q', n', q, n). \end{aligned}$$

Letting $G(\alpha) = \delta(1 - Eq + \Delta q(\alpha))$, an equilibrium is a fixed point of this function. To prove that this function is continuous, we apply the above lemma. For that purpose, it is convenient to represent the trading set by functions, as follows. Let $g_{11}(q, n, q', n'; \alpha) = \min(W(q, n; \alpha) - W(q', n'; \alpha), q' - q)$, $g_{10}(q, n, q', n'; \alpha) = \min(W(q, n; \alpha) - W(q', n'; \alpha), G_P(\pi, n, \pi', n'; \alpha))$ and define analogously g_{01} and g_{00} . Generically, the zeros of these functions will be regular points and since μ is continuous, the set of zeros will be μ null sets. Applying the above lemma $\Delta q(\alpha)$ will be continuous and so will function G . Since $G: [0, 1] \rightarrow [0, 1]$, there exists a fixed point.

Sketch of proof of equilibria with an ex ante mechanism of intertemporal reallocation. It is easy to see that the participation of patient agents is given by a threshold q_P such that for all $q \geq q_P$ all patient agents trade through intermediation. Let

$$\Pi_I(\beta, q_P) = \mu_I \int_{A_I(\beta, q_P)} \pi(q) dF(q, n),$$

where

$$A_I(\beta, q_P) = \{(q, n) \mid B_I(q, n; \beta, q_P) \geq 0\}.$$

By repeated application of Lemma 2, Π_I is continuous. Let

$$\Pi_P(q_P) = \mu_P \int_{q \geq q_P} \pi(q) dF(q, n)$$

and

$$F(\beta, q_P) = \Pi_I(\beta, q_P)\beta - \frac{\delta}{1-\delta} \Pi_P(q_P).$$

F is increasing in β and by Lemma 2 it is continuous. For large β , $F(\beta, q_P) > 0$ and for $\beta \leq \beta_I$ it is negative. Hence for each q_P there exists a unique value of β that makes $F(\beta, q_P) = 0$. Denote this by function $Z(q_P)$. Applying Berge it follows that this function is also continuous. Finally, define function $\Gamma(q_P)$ to be the unique value q' such that

$$B_P(q'; A_I(Z(q_P), q_P)) = 0$$

and

$$\begin{aligned} q' = 1 & \quad \text{if } B_P(q'; A_I(Z(q_P), q_P)) > 0 \quad \text{for all } q' \\ q' = 0 & \quad \text{if } B_P(q'; A_I(Z(q_P), q_P)) < 0 \quad \text{for all } q'. \end{aligned}$$

This is well defined since B_P is strictly increasing in q . Function Γ maps the unit interval into itself. A fixed point for Γ is an equilibrium. Applying Lemma 2, it can be shown that Γ is continuous and thus a fixed point exists. ■

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