

## On the Effects of Barriers to International Investment

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### ABSTRACT

A simple model is presented in which it is costly for domestic investors to *hold* foreign assets. The implications of the model for the composition of optimal portfolios at home and abroad are derived. It is shown that all foreign assets with a beta larger than some beta  $\beta^*$  plot on either one of two security market lines. Some foreign assets with a beta smaller than  $\beta^*$  are not held by domestic investors even if their expected return is increased slightly.

WHILE IT IS OBVIOUSLY not true that asset markets are completely segmented between countries, there is evidence of barriers to international investment. Although reality seems to lie in that grey area between complete segmentation and no segmentation at all, most international asset pricing models are concerned with the extreme case of no barriers to international investment. No effort seems to have been made to study the effect on portfolio choice of such barriers, which make it costly to *hold* foreign securities, as opposed to domestic securities, but which do not, in general, render international diversification so onerous that investors avoid foreign securities completely.<sup>1</sup> Casual empiricism suggests that models without barriers to international investment should be suspect; those models cannot explain why it appears that in every country investors, on average, hold more domestic securities than would be required if they held the world market portfolio.<sup>2</sup>

This paper constructs a model of international asset pricing in which there is a cost associated with holding—either long or short—risky foreign securities. For the sake of simplicity, in most of the paper we assume that while domestic investors face barriers to international investment, foreign investors face no such barriers. It turns out that nothing significant is lost with such an assumption. The main conclusions of this paper for the case in which foreign investors face no barriers to international investment are:

1. In the presence of barriers to international investment, some risky foreign

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<sup>1</sup> The most important exception is Black [3]. Stapleton and Subrahmanyam [7] have a numerical example of incomplete segmentation. See also Adler and Dumas [2].

<sup>2</sup> Note that the fact that investors do not hold the world market portfolio is not very interesting. What is important is that everywhere investors have a bias towards domestic stocks. It is true, however, that this bias is not equally strong in all countries.

assets can be nontraded, in the sense that they are not held by domestic investors and would not be held if their expected return changed slightly.

2. In each country, all investors hold the same portfolio of risky assets.
3. Traded assets plot on well-defined security market lines. There is one security market line (SML) on which all domestic assets plot. Risky foreign assets held long (short) by domestic investors plot on an SML which lies above (below) and is parallel to the SML for risky domestic assets.
4. Nontraded risky assets plot between the two SML's for risky foreign assets.
5. For investors who face barriers to international investment, the world market portfolio is inefficient in that there cannot exist a mutual fund for domestic investors which would make them indifferent between choosing an appropriate combination of the safe asset, the mutual fund, and the world market portfolio, or holding their own portfolio.
6. There exists a (finite) beta  $\beta^*$  such that all assets which have a beta larger than  $\beta^*$  are traded.

As in Black [3], proportional taxes are used to model barriers to international investment. However, the only result we have summarized which also holds for the Black model is that in each country, all investors hold the same portfolio of risky assets. In the Black model, no *risky* asset can be nontraded.<sup>3</sup> We obtain very different results from those of the Black model because in our model an investor pays a tax proportional to the *absolute* value of his holdings of risky foreign assets, whereas in the Black model an investor has to pay taxes in proportion to his *net* holdings of risky foreign assets. The barriers to international investment in the Black model hinder *net* investment in risky foreign assets, whereas in our model they make it more difficult to *hold* risky foreign assets. In our model, if an investor pays a tax  $\theta$  on each dollar held long in foreign assets, he also pays a tax  $\theta$  on each dollar held short in foreign assets.

Barriers to international investment can take a variety of forms, some of them nonpecuniary, and taxes are a way, albeit imperfect, to represent them. The barriers to international investment we try to model are of a different type from those modeled by Black. In the Black model, an increase in barriers to international investment will never induce domestic investors to abstain from holding foreign risky securities. If the barriers to international investment modeled by Black were very high, one would not observe domestic investors holding few foreign securities, but rather would observe them holding large amounts of foreign securities short. One does not observe that whenever domestic investors have small long holdings in a foreign country, they also have large short holdings. This suggests that there must indeed exist barriers to international investment which make it difficult, for domestic investors, to hold—either long or short—foreign risky securities. The present paper tries to model such barriers.

Because exchange rate risks are irrelevant to our argument, we use a framework in which exchange rates do not appear at all. Introducing exchange rates in our model would *not* change our results; the effect of barriers to international

<sup>3</sup> In the Black model, it is possible for an investor to have zero holdings in a foreign risky asset. However, an infinitesimal change in the expected return of that asset would induce that investor to hold it—either short or long. In the Black model, the safe asset can be nontraded. The same result would obtain in our model if we introduced a tax on borrowing or lending abroad.

investment would be the same as in this study so long as those barriers to international investment were of a type which in the limit can produce complete segmentation. Formally, the portfolios which obtain in this paper would obtain in a world in which there is only one good, in which there is a safe real bond in each country, and in which there are neither transportation costs nor tariffs.

Finally, the model presented here is not a general equilibrium model, in the sense that the barriers to international investment are given and no attempt is made to explain those barriers. To the extent that those barriers correspond to taxes, no attempt is made to explain how the revenue from those taxes is spent.

## I. The Model

Throughout this paper, we assume that there are only two countries, the domestic country  $D$  and the foreign country  $F$ . Let an investor  $k$  be a domestic (foreign) investor if we write  $k \in D$  ( $k \in F$ ). An asset  $i$  is a domestic (foreign) asset if we write  $i \in D$  ( $i \in F$ ). We assume that domestic investors face a very simple form of barriers to international investment, whereas foreign investors face no barriers to international investment. If a domestic investor  $k$  holds a foreign risky asset  $i$  long, his return is  $\tilde{R}_i - \theta$ , where  $\tilde{R}_i$  is the return of asset  $i$  for a foreign investor and  $\theta$  is the tax rate which represents barriers to international investment, whereas if he holds asset  $i$  short and keeps the proceeds in cash, his return is  $-(\tilde{R}_i + \theta)$ . Unlimited short-sales with full use of the proceeds are permitted. A portfolio which consists of one dollar sold short in one foreign security, with the proceeds of that short-sale invested long in another foreign risky asset, pays a tax of  $2\theta$ .<sup>4</sup> The barrier to international investment can usefully be thought of as a penalty for holding foreign shares long or short. Both domestic and foreign investors can buy or sell bonds which have a safe rate of return  $R$ . If domestic investors borrow or lend abroad at the safe rate, they do not pay a tax on their holdings of foreign bonds.<sup>5</sup>

Each investor is assumed to maximize a utility function which depends positively on expected end-of-period wealth and negatively on the variance of end-of-period wealth. With that assumption, the investor acts to minimize the variance of the return of his portfolio under the constraint that the expected return of his portfolio must be no less than an exogenously given return,  $\bar{R}^k$ . By convention, there are  $N$  risky assets and  $n$  risky domestic assets.  $\bar{R}$  is the  $N \times 1$  vector of expected returns of risky assets and  $V$  the  $N \times N$  variance-covariance matrix of returns of risky assets. Let  $w^k$  be the  $N \times 1$  vector of fractions of his wealth investor  $k$  holds long in risky assets. If  $w_i^k = 0$ , this means that investor  $k$  does

<sup>4</sup> In the Black model, because there is a subsidy on short-sales, the investor would not pay taxes on the portfolio we just described.

<sup>5</sup> Stulz [8] presents a more general model in which this assumption does not hold. Black [3] shows that if a tax is imposed on borrowing and lending abroad, it is possible that neither borrowing nor lending occurs between countries. Furthermore, when there is a tax on borrowing and lending abroad, it will never be the case that some domestic investors borrow abroad, whereas other domestic investors lend abroad. Either some domestic investors borrow abroad and none lend abroad, or vice versa. These results also hold in Stulz [8]. Introducing a tax on borrowing and lending abroad would complicate the present paper without offering any additional insight.

not hold a positive amount of risky asset  $i$ . Similarly, let  $v^k$  be the  $N \times 1$  vector of fractions of his wealth the investor holds short in risky assets.  $v_i^k > 0$  means that the investor  $k$  holds short risky asset  $i$  for an amount equal to  $v_i^k W^k$ , where  $W^k$  is the investor's wealth. The portfolio of risky assets of the investor is given by  $(w^k - v^k)$ . If  $e$  is an  $N \times 1$  vector which has zeros in its first  $n$  rows and ones everywhere else, the taxes a domestic investor  $k$  has to pay are equal to  $(w^k + v^k)' e \cdot \theta \cdot W^k$ , where a prime denotes a transpose. The elements of  $w^k$  and  $v^k$  must be nonnegative to guarantee that the investor cannot construct a portfolio which would transform taxes into subsidies. It will be convenient to define  $\underline{l}$  as an  $N \times 1$  vector of ones. It follows that the problem of investor  $k$  is to minimize the variance of his portfolio subject to a constraint on the expected rate of return of his portfolio and nonnegativity constraints on  $w^k$  and  $v^k$ .<sup>6</sup>

$$\text{Min } \frac{1}{2} (w^k - v^k)' V (w^k - v^k)$$

so that the following constraints are satisfied:

$$(w^k - v^k)' \bar{R} - (w^k + v^k)' e \cdot \theta + [1 - (w^k - v^k)' \underline{l}] R \geq R^k \quad (1)$$

$$w^k \geq 0 \quad (2)$$

$$v^k \geq 0 \quad (3)$$

The left-hand side of Expression (1) corresponds to the expected return of the portfolio of investor  $k$ , which is defined as the sum of (a) the expected return of his holdings of risky assets in the absence of barriers to international investment; (b) the total cost to the investor of the barriers to international investment, which is proportional to the absolute value of the investor's holdings of foreign risky assets; and (c) his holdings of safe bonds. If  $L^k$  is the Lagrangean function which corresponds to the investor's optimization problem and if  $\lambda^k$  is the multiplier associated with the constraint given by (1), then the investor's portfolio has to satisfy the following first-order conditions:

$$\frac{\partial L^k}{\partial w_i^k} = V (w^k - v^k) - \lambda^k \{ \bar{R} - R \cdot \underline{l} - \theta \cdot e \} \geq 0 \quad (4)$$

$$\frac{\partial L^k}{\partial v_i^k} = -V (w^k - v^k) + \lambda^k \{ \bar{R} - R \cdot \underline{l} + \theta \cdot e \} \geq 0 \quad (5)$$

$$(w^k)' \frac{\partial L^k}{\partial w_i^k} = 0 \quad (6)$$

$$(v^k)' \frac{\partial L^k}{\partial v_i^k} = 0 \quad (7)$$

To obtain the first-order conditions for the portfolio of a foreign investor, set  $\theta$

<sup>6</sup> Note that the optimization problem of the investor is very similar to the problem faced by investors in the literature dealing with transaction costs, except that here the initial allocation does not matter. Smith and Milne [5] derive equilibrium relationships in a model with transaction costs. For a discussion of optimal portfolios in the presence of transaction costs and references to the literature, see Abrams and Karmarkar [1].

equal to zero in (4) and (5). If we use  $\underline{V}_i$  for the  $i$ -th row of  $\underline{V}$ , Expressions (4) and (5) can be rewritten as:

$$\lambda^k(\bar{R}_i - R + \theta) \geq \underline{V}_i(w^k - v^k) \geq \lambda^k(\bar{R}_i - R - \theta) \tag{8}$$

where  $\underline{V}_i(w^k - v^k)$  is the covariance between the return on asset  $i$  and the return on the investor's portfolio of risky assets. Expression (8) has to be satisfied, for  $\theta > 0$ , for all risky foreign assets in the portfolio of domestic investor  $k$ . For  $\theta = 0$ , Expression (8) must be satisfied for all assets  $i$ , for all foreign investors, and for all domestic assets  $i$ , for all domestic investors. If the tax rate for an asset is equal to zero, i.e.,  $\theta = 0$ , (8) reduces to:

$$\lambda^k(\bar{R}_i - R) = \underline{V}_i(w^k - v^k) \tag{9}$$

Expression (8) completely characterizes asset demands in our model. The implications of Expression (8) are discussed in detail in the next three sections.

## II. Asset Demands and Non-Traded Assets

In this section, we prove two results. The first is that the proportion in which two risky assets are held is the same for each domestic investor. The second result is that for a given vector of expected excess returns and variance-covariance matrix of returns, some foreign assets can be nontraded. A foreign asset is nontraded if no domestic investor currently holds a nonzero number of shares of that asset and if no domestic investor would hold a nonzero number of shares of that asset if its expected return increased or decreased slightly.

First, we show that all domestic investors hold risky assets in identical proportions. (The same holds for foreign investors, but for them it is trivial, as they do not face any barriers to international investment.) From first-order conditions (4) to (7), it follows that both inequalities in (8) can hold strictly only if the investor does not hold that asset, i.e.  $w_i = v_i = 0$ . If we look only at the assets which satisfy (8) with one equality and one strict inequality and at the assets which satisfy (9), we can completely characterize an investor's portfolio. If we divide the first-order condition which an asset held in nonzero amount satisfies by the first-order condition a domestic asset  $j$ , such that  $\bar{R}_j \neq R$ , satisfies, we can write:

$$\frac{\underline{V}_i(w^k - v^k)}{\underline{V}_j(w^k - v^k)} = \frac{(\bar{R}_i - R - p\theta)}{\bar{R}_j - R} \tag{10}$$

where  $p = 1$  if the asset is a foreign asset held long,  $p = 0$  if the asset is a domestic asset, and  $p = -1$  if the asset is a foreign asset held short. Because the right-hand side of (10) contains no taste variable, Equation (10) must hold for all domestic investors and for all assets  $i$ . (10) defines a system of  $N - 1$  equations with  $N - 1$  unknowns, where the unknowns are the ratios  $(w_i^k - v_i^k)/(w_j^k - v_j^k)$  for any investor  $k$ . It follows necessarily that the ratios  $(w_i^k - v_i^k)/(w_j^k - v_j^k)$ , for all  $i$ 's, must be the same for all domestic investors. This completes the proof of our result.

Our first result is important here because it implies that an asset which is nontraded for one domestic investor is nontraded for *all* domestic investors. We call a nontraded asset for domestic investor  $i$  an asset (a) of which the investor

holds no share and (b) which the investor would not hold if the expected return of that asset increased or decreased slightly. For an asset to be nontraded, both inequalities in (8) have to hold strictly. We have just proved that if one inequality does not hold strictly in (8) for *one* domestic investor, it holds with equality for all domestic investors. It follows that a nontraded asset will be nontraded for all domestic investors, which means that whether an asset is traded or not has nothing to do with an investor's preferences. If both inequalities in (8) hold strictly, there is a number  $\epsilon$  such that if the expected return on the stock is either  $\bar{R}_i + \epsilon$  or  $\bar{R}_i - \epsilon$ , those inequalities still hold strictly.

The fact that it is possible for nontraded assets to exist can be established easily. Choose the variance-covariance matrix of returns to be such that, in the last  $N^*$  rows, the only nonzero elements are the diagonal elements. For foreign assets, Expression (8) can be rewritten as:

$$\lambda^k(\bar{R}_i - R + \theta) \geq \sigma_i^2(w_i^k - v_i^k) \geq \lambda^k(\bar{R}_i - R - \theta) \quad (11)$$

where  $\sigma_i^2$  is the variance of the return of the  $i$ -th asset. Suppose that  $\bar{R}_i - R$  is approximately equal to zero. In this case, the investor will not hold the stock long, because the second inequality in (11) does not hold strictly, implying that  $w_i^k$  is negative, as  $\theta$  is greater than zero. The investor will not hold the stock short, because this implies that  $v_i^k$  is negative. It follows that both inequalities hold strictly.

We have shown that for a given vector of expected excess returns and a given variance-covariance matrix of returns, it is possible for some assets to be nontraded and that if one asset is nontraded for a domestic investor, it will be nontraded for all domestic investors. Only empirical research can solve the question of whether or not some assets will be nontraded for some investors.

The *theoretical* result that nontraded assets *can* exist has, however, far-reaching implications. The first is that if there exist nontraded assets, the world portfolio of stocks cannot possibly be an efficient portfolio of stocks for all investors. In the next section, we will discuss the implication of that fact for what is usually thought of as the security market line. The result here that if the tax rate  $\theta$  is positive the world market portfolio is not an efficient portfolio has the strong implication that if a mutual fund sells shares of the world market portfolio, no domestic investor wants to buy them. In our model, contrary to the model of Black [3], if a mutual fund sells claims to the world market portfolio, it is costly to undo its actions, (for instance, selling short an asset held long by the mutual fund involves paying the tax twice) and so investors are never indifferent between holding shares in that mutual fund and in some other portfolio, or holding their own portfolio of risky assets. In other words, contrary to the Black model, the world market portfolio cannot belong in a linear combination of portfolios which would produce an efficient portfolio for domestic investors. If there are nontraded assets, it is not true that adding foreign assets to a portfolio of domestic assets is necessarily a good thing. Finally, whereas proxies for the market portfolio are generally criticized for not including more risky assets, the theoretical possibility exists that, for some tests, some of those proxies can include too many assets.<sup>7</sup>

<sup>7</sup> See Roll [4] for a thorough discussion of the problems posed by the fact that we cannot observe the true market portfolio.

III. Is there a Security Market Line?

We have shown that some risky assets can be nontraded and that consequently the market portfolio is not even part of a linear combination of portfolios which yields an efficient portfolio for domestic investors. In this section, we show that a plot of expected returns of risky assets with respect to their betas computed using the world market portfolio is not arbitrary. In particular, all foreign traded assets plot along one of two straight lines whereas nontraded assets plot between those straight lines.

Let us look back at Expression (8) which gives bounds which a portfolio of risky assets must satisfy. This expression can be rewritten as (remember that if investor  $k$  is a foreign investor,  $\theta = 0$ ):

$$\lambda^k(\bar{R}_i - R + \theta) - \lambda^k Q_i^k = V_i(w^k - v^k) = \lambda^k(\bar{R}_i - R - \theta) + \lambda^k q_i^k \quad (12)$$

where  $Q_i^k$  and  $q_i^k$  are nonnegative numbers chosen so that the inequalities in (8) hold with equality signs.  $Q_i^k$  and  $q_i^k$  are not observable, but they facilitate considerably the derivation of our results.  $Q_i^k$  and  $q_i^k$  must take values so that they satisfy:

$$q_i^k + Q_i^k = 2\theta \quad i \in F \quad (13)$$

$$q_i^k = Q_i^k = 0 \quad i \in D \quad (14)$$

Let  $\underline{q}^k$  be a  $N \times 1$  vector whose representative element is  $q_i^k$ . We can write:

$$\underline{V}(w^k - v^k) = \lambda^k(\bar{\underline{R}} - \underline{l}R - \theta \underline{e}) + \lambda^k \underline{q}^k \quad (15)$$

Equation (15) is a restatement in matrix form of the right-hand side of the first equality in (12), for all  $i$ 's. Define  $w^s$  as an  $N \times 1$  vector whose element  $w_i^s$  is equal to the fraction of world wealth  $W^w$  supplied in the form of risky asset  $i$ . If  $\lambda^k W^k$  is equal to  $T^k$ ,  $T^d(T^f)$  is equal to the sum of the  $T^k$ 's for  $k \in D$  ( $k \in F$ ),  $\pi^k$  is equal to  $T^k/T^d$  and  $\gamma^d$  is equal to  $T^d/(T^d + T^f)$ , we get, after some rearranging:

$$\underline{V}w^s W^w = (T^d + T^f)(\bar{\underline{R}} - \underline{l}R - \gamma^d \theta \underline{e} + \gamma^d \sum_{k \in D} \pi^k \underline{q}^k) \quad (16)$$

Equation (16) assumes that all markets for risky securities are in equilibrium. The last term on the right-hand side of (16) which we now write  $\gamma^d \underline{q}^d$ , is a weighted average of the  $\underline{q}^k$ 's across all investors. If Equation (16) is premultiplied by  $w^s$ , the left-hand side of the resulting expression is equal to the variance of portfolio  $w^s$ , which we write  $\sigma_w^2$ , multiplied by  $W^w$ :

$$\sigma_w^2 W^w = (T^d + T^f)\{\bar{R}_m - R - \theta_m + q_m\} \quad (17)$$

where  $\bar{R}_m$  is equal to the expected return on portfolio  $w^s$ ,  $\theta_m$  is equal to  $\gamma^d$  times the amount of taxes domestic investors would have to pay on one dollar invested in portfolio  $w^s$ , and  $q_m$  is equal to  $(w^s)' \underline{q}^d \gamma^d$ . Equation (17) is used to eliminate  $(T^d + T^f)$  in (16):

$$\beta^m[\bar{R}_m - R - \theta_m + q_m] = \bar{\underline{R}} - \underline{l}R - \gamma^d \theta \underline{e} + \gamma^d \underline{q}^d \quad (18)$$

Equation (18) is the fundamental asset pricing equation of this paper.  $\beta_i^m$  is the beta of common stock  $i$  computed using the *world* market portfolio, i.e.  $\text{Cov}(\bar{R}_i, \bar{R}_m)/\text{Var}(\bar{R}_m)$ . With no barriers to international investment, (18) reduces to the usual Sharpe-Lintner pricing relationship.<sup>8</sup> We now look successively at the pricing of domestic stocks and foreign stocks when the tax rate  $\theta$  is greater than zero.

Domestic stocks are equally easy to hold for foreign and domestic investors, because for foreign investors there are no barriers to international investment. We can rewrite (18) as it applies to domestic common stocks:

$$\beta_i^m[\bar{R}_m - R - \theta_m + q_m] = \bar{R}_i - R \quad i \in D \quad (19)$$

If the capital asset pricing model held for domestic stocks,  $\theta_m$  and  $q_m$  would be equal. With our assumptions,  $\theta_m$  is a strictly positive number, whereas  $q_m$  is a nonnegative number. However,  $q_m$  can be larger than  $\theta_m$ . If the value of foreign stocks is at least twice the value of foreign stocks which are nontraded, then  $q_m$  will be smaller than  $\theta_m$ . In that case, domestic common stocks will plot on a security market line which has a smaller slope than the security market line which corresponds to the Sharpe-Lintner pricing relationship. If most foreign stocks are nontraded, no such statement can be made without restricting the variance-covariance matrix of asset returns. For instance, for a diagonal variance-covariance matrix, it can be shown that  $\theta_m$  will always be larger than  $q_m$ . To interpret  $q_m$ , it is useful to ask the following question: suppose that *one* domestic investor is given the choice of having either the barrier on positive holdings or the barrier on negative holdings of foreign stocks removed, which barrier would he want to get rid of? As long as the investor prefers to have the barrier on positive holdings removed, which is not necessarily the case because of the distortions introduced by the barriers on foreign investment,  $q_m$  will be smaller than  $\theta_m$ . In that case, the Sharpe-Lintner model will overpredict the returns of domestic risky assets and the expected prediction error will be an increasing function of beta.

The asset pricing relationship for foreign risky assets is:

$$\beta_i^m[\bar{R}_m - R - \theta_m + q_m] + \gamma^d\theta - q_i^d = \bar{R}_i - R \quad i \in F \quad (20)$$

Whenever two risky foreign assets are held long by domestic investors, the expected returns of those two risky assets can differ only to the extent that the beta of those two assets differ, as for all risky foreign assets held long by domestic investors,  $q_i^d = 0$ . It follows that the expected returns of foreign risky assets held long by domestic investors must be a linear function of beta. For all risky foreign assets held short by domestic investors,  $q_i^d = 2\gamma^d\theta$ . This implies that all risky foreign assets held short by domestic investors must plot on a security market line, but that security market line lies below the security market line for domestic risky assets, whereas all risky foreign assets held long by domestic investors plot on a security market line which lies above the security market line for domestic risky assets.

<sup>8</sup> If foreign investors face barriers to international investment or if the tax on short holdings is different from the tax on long holdings, this affects  $\theta_m$  and  $q_m$ .

The two security market lines for foreign traded risky assets will be parallel to the security market line for domestic risky assets. Nontraded assets will plot *between* the two security market lines for foreign risky assets. Figure 1 illustrates those results for the case in which  $\theta_m$  is larger than  $q_m$ . Asset 1 is a domestic risky asset; asset 2 is a foreign risky asset held long; asset 3 is a foreign risky asset held short; asset 4 is a foreign nontraded asset.

Suppose that a researcher assumes that the Sharpe-Lintner pricing relationship holds when, in fact, Equation (20) holds. In that case, define  $\alpha_i$  as the difference between the true expected return of risky asset  $i$  and the expected return which corresponds to the Sharpe-Lintner model:

$$\alpha_i = \gamma^d \theta - q_i^d + \beta_i^m (q_m - \theta_m) \quad i \in F \tag{21}$$

If  $q_m$  is smaller than  $\gamma^d \theta$ , it immediately follows from (21) that traded foreign assets held long by domestic investors will have alphas which can be expressed as a decreasing function of beta. Zero-beta foreign assets held long by domestic investors will have positive alphas. Zero-beta foreign assets which are nontraded will have alphas between  $\gamma^d \theta$  and  $(-\gamma^d \theta)$ . Finally, zero-beta foreign assets held short will have negative alphas. All *traded* assets, either foreign or domestic, will have alphas which can be expressed as a decreasing function of beta.

The results of this section have important empirical implications. First, note that if there are barriers to international investment for domestic investors, but not for foreign investors, a portfolio of domestic risky assets with a beta of one will in general have an expected return different from the expected return of the world market portfolio. Furthermore, if it can be shown that a linear relationship exists for all assets of a country between their beta and their expected return, then no asset of that country is a nontraded asset. If there are no barriers to

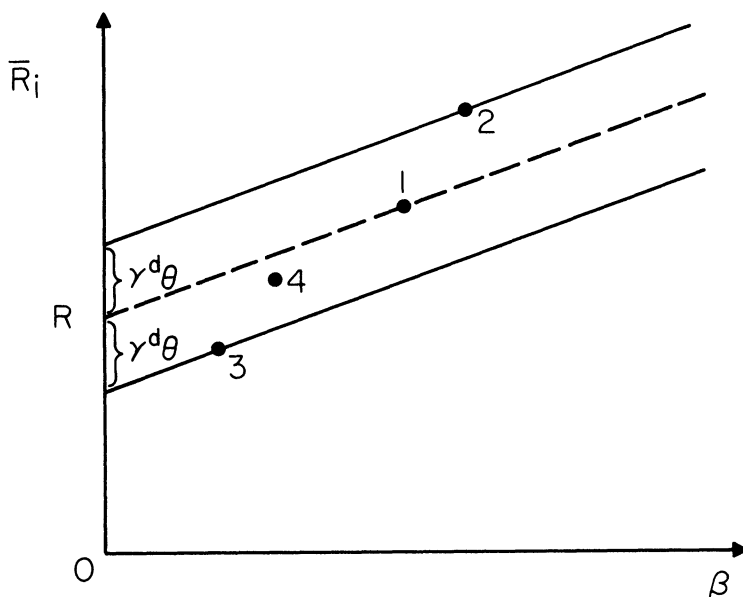


Figure 1. Security market lines

international investment for domestic assets, a domestic zero-beta portfolio would have an expected return equal to the rate of interest. Because a foreign zero-beta portfolio can include nontraded assets, its expected return can be greater than or smaller than the rate of interest.

IV. Which Assets are Non-Traded?

Section III showed that nontraded assets will plot between two security market lines which we described. It would be useful to find out more about nontraded assets. If we know which assets are likely to be nontraded by domestic investors, we have information from which to estimate the security markets lines. Furthermore, we can characterize more precisely efficient portfolios.

Aggregating Expression (8) across domestic investors, we get:

$$\bar{R} - R.\underline{l} + \theta.e \geq \frac{1}{T^d} \sum_{k \in d} (w^k - v^k) W^k \geq \bar{R} - R.\underline{l} - \theta.e \tag{22}$$

For a nontraded asset, both inequalities in (22) must hold strictly. Define  $\Omega^d$  as the proportion of domestic wealth  $W^d$  invested in risky assets:

$$\Omega^d = \frac{1}{W^d} \sum_{v_i} \sum_{k \in D} (w_i^k - v_i^k) W^k \tag{23}$$

Let  $(w^d - v^d)$  be a portfolio of risky assets whose weights sum up to one and such that the ratio of holdings of any two assets is the same as the ratio in which the domestic country holds these two risky assets. For a nontraded asset, Expression (22) can be rewritten as:

$$\bar{R}_i - R + \theta > \left( \frac{\Omega^d W^d}{T^d} \right) Y_i(w^d - v^d) > \bar{R}_i - R - \theta \tag{24}$$

$Y_i(w^d - v^d)$  is equal to the covariance of the  $i$ -th asset with the *domestic* portfolio of risky assets. We can define  $\Omega^f$  as the fraction of foreign wealth  $W^f$  invested in risky assets and  $(w^f - v^f)$  as the portfolio of risky assets for foreign investors defined in the same way as  $(w^d - v^d)$ . With this notation, Expression (9) implies that if we write  $\text{Cov}(\tilde{R}_i, \tilde{R}^F)$  for  $Y_i(w^f - v^f)$ , the following relationship must hold:

$$\text{Cov}(\tilde{R}_i, \tilde{R}^F) = \frac{T^f}{\Omega^f W^f} (\bar{R}_i - R) \tag{25}$$

Equation (25) can be used to eliminate the expected excess return in (24):

$$\theta > \left( \frac{\Omega^d W^d}{T^d} \right) \text{Cov}(\tilde{R}_i, \tilde{R}^D) - \left( \frac{\Omega^f W^f}{T^f} \right) \text{Cov}(\tilde{R}_i, \tilde{R}^F) > -\theta \tag{26}$$

where  $i$  is a nontraded asset for domestic investors, and  $\text{Cov}(\tilde{R}_i, \tilde{R}^D)$  is  $Y_i(w^d - v^d)$ . From Expression (26), it follows that if there is an asset which is correlated neither with the domestic portfolio nor with the foreign portfolio of risky assets, that asset will necessarily be nontraded. A risky asset which is correlated neither with the portfolio of risky assets held by domestic investors, nor with the one held by foreign investors, must be an asset with a beta of zero. Looking at Expression (26), one would think that a risky asset with a beta different from

zero in absolute value by some small number  $\epsilon$  would still be a nontraded asset. It is shown in the appendix that (26) can be rewritten in such a way that it depends on the covariance of the asset being considered with the market portfolio and a constant. It immediately follows from that result that there must exist a beta  $\beta^*$  such that all assets with a larger beta must be traded. Because beta is a measure of the risk of an asset in a world without barriers to international investment, this result means that such barriers decrease trade in the least risky assets. Those assets do not provide an expected return large enough to offset the cost of holding them due to the barriers.

The fact that assets with low betas are those likely to be nontraded is important for empirical research. First, this result should make us very cautious about using portfolios which have a large proportion of assets with low beta in tests of international asset pricing models.<sup>9</sup> Secondly, this result suggests that, when estimating security market lines, it might be worthwhile to test whether a security market line estimated using only assets with a high beta has a different slope from one estimated using only assets with a low beta. Should the slope of these two security market lines be different, this could be used as evidence of the existence of barriers to international investment. Finally, one should be very careful in devising techniques to form a portfolio of foreign stocks. Buying the market portfolio in a foreign country might just be buying a highly inefficient portfolio for domestic investors.

### V. Concluding Remarks

In this paper, we constructed a model of international asset pricing such that, for domestic investors, it is equally costly to hold foreign securities long or short. It can easily be shown that the results we obtained hold as long as barriers to international investment make it costly for a domestic investor to hold the same foreign security simultaneously long and short. Our results generalize in a natural way if foreign investors also face barriers to international investment.<sup>10</sup>

The model presented here holds if barriers to international investment can be represented as taxes on the *absolute* value of an investor's holdings of risky foreign assets. The Black model holds if barriers to international investment correspond to taxes on the *net* value of an investor's holdings of foreign risky assets. In the Black model, investors are indifferent between holding their portfolio of risky assets or their portfolio plus one foreign share held simultaneously long and short, whereas in our model investors offered this choice would never be indifferent. In our model, short-sales do not entail a subsidy. Both our model and the Black model offer well-defined null hypotheses against which the hypothesis of no barriers to international investment can be tested. Because earlier tests of models of international asset pricing were plagued by the lack of a well-defined null hypothesis,<sup>11</sup> our model leads to more powerful tests of models without barriers to international investment.

<sup>9</sup> For instance, Black [3] suggested looking at the minimum variance zero-beta portfolio. Clearly, in our model, that portfolio could have an expected return equal to the rate of interest in the presence of barriers to international investment.

<sup>10</sup> Stulz [8] contains a very general version of the present paper.

<sup>11</sup> See Solnik [6] and the comments following his paper.

Let  $h^f$  and  $h^d$  be two portfolios such that:

$$w^d - v^d + h^d = w^f - v^f + h^f = w^s \quad (\text{A.1})$$

It follows that  $(h^d)'l = (h^f)'l = 0$ . It must be true that:

$$\Omega^d(w^d - v^d)W^d + \Omega^f(w^f - v^f)W^f = w^s W^w \quad (\text{A.2})$$

From (A.2):

$$-\Omega^d h^d W^d = \Omega^f h^f W^f \quad (\text{A.3})$$

From (9) and (A.3), after setting  $-h^d = \underline{h}$ :

$$w^f - v^f = w^s + \left( \frac{\Omega^d W^d}{\Omega^f W^f} \right) \underline{h} = V^{-1} \left( \frac{T^f}{\Omega^f W^f} \right) (\bar{R} - Rl) \quad (\text{A.4})$$

Substituting (17) and (18) in (9), we get:

$$\Omega^f = \frac{T^f}{W^f} \frac{W^w}{(T^d + T^f)} + \frac{T^f}{W^f} l' V^{-1} (\gamma^d \underline{e} \theta - q^d \gamma^d) \quad (\text{A.5})$$

Solving (A.4) for  $\underline{h}$  yields, after using (18):

$$\underline{h} = \frac{T^f}{\Omega^d W^d} \{-l' V^{-1} (\gamma^d \underline{e} \theta - q^d \gamma^d) w^s\} + \frac{T^f}{\Omega^d W^d} \{V^{-1} (\gamma^d \underline{e} \theta - q^d \gamma^d)\} \quad (\text{A.6})$$

Substituting (A.6), in (26) using (A.1) and (A.5) yields an expression which can be written:

$$\theta > b \text{Cov}(\tilde{R}_i, \tilde{R}_m) + d[\gamma^d \theta - q_i^d \gamma^d] > -\theta \quad (\text{A.7})$$

where  $b$  and  $d$  can be zero only if  $\theta = 0$ .

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