

**MACROECONOMIC TIME-SERIES,  
BUSINESS CYCLES AND MACROECONOMIC POLICIES**

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**I. INTRODUCTION AND SUMMARY**

The decomposition of real variables into a trend or growth, a cyclical, and a seasonal component has a long history in macroeconomics. The cyclical part is generally considered to be of special interest, because it is believed to dominate short-run fluctuations in real activity and to respond directly to monetary and fiscal policy measures. Unfortunately, the cyclical component, as the others, is not directly observable. In order to approximate the business cycle, it has become common practice in empirical work to correct for long-run growth and seasonals by including a linear time trend, and where appropriate, seasonal dummy variables in the respective regressions.<sup>1</sup> Nelson/Plosser (1982) show that the growth component is better approximated by a stochastic model, specifically a random walk, for a large number of annually measured time-series in the United States. If the growth component is approximated by a random walk, the business cycle component of real variables turns out to be much smaller

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<sup>1</sup>See, for instance, Barro (1978), Lawrence (1983), Lucas (1973), Mishkin (1982), Sargent (1976), and Taylor (1979).

than if the growth component is erroneously approximated by a time trend. This follows from the fact that if one regresses a random walk on a time trend, the residuals of the regression exhibit strong and positive autocorrelations, which are a pure statistical artifact exclusively determined by sample size. Nelson/Plosser (1982) show that, empirically, the decomposition of time-series of real variables using a stochastic trend implies that fluctuations in real variables are dominated by changes in the permanent rather than in the transitory, i.e., business cycle, component.

In the first part of this paper, we provide some evidence of the robustness of the results of Nelson/Plosser (1982). Their study used only annual data and focused only on U.S. time-series. We use monthly and quarterly data and look at several countries. We find that in general our results corroborate the results of Nelson/Plosser (1982). We show that the cyclical component, if it exists at all, can be well approximated by a low-order moving average process. According to our results, the business cycle does not persist beyond one year.

The empirical part of the paper shows that changes in the output trend play a crucial role in macroeconomic fluctuations. Traditionally, explaining the trend of output has been the domain of growth theory, while business cycle theory has tried to explain deviations of output from its trend. Our empirical work shows that, to be empirically relevant, a theory of macroeconomic fluctuations must explain both changes in the trend of output and deviations of output from its trend. In the second part of the paper, we offer a model which provides an explanation for changes in output which is consistent with our empirical analysis.<sup>2</sup> The model focuses on the fact that households can choose the distribution of output when they can invest among many technologies. It is shown that a change in technologies or monetary policy brings about a change in how households invest their wealth and, hence, in the distribution of output. The major lesson of our model is, therefore, that changes in the trend of output and, hence, macroeconomic fluctuations, can be caused both by changes in real variables which are usually considered by growth theory, i.e., technologies, and by changes in variables which are usually considered by business cycle theories, i.e., monetary policy. The paper ends with some concluding remarks.

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<sup>2</sup>This research builds on recent models which focus on the real business cycle; see, for instance, Black (1979), Long/Plosser (1981), King/Plosser (1982), and Kydland/Prescott (1981).

## II. EMPIRICAL FINDINGS

In this part, a number of empirical tests are carried out which allow important insights into the stochastic characteristics of the magnitudes under investigation. The presentation is organized as follows: First, the statistical frameworks used to decompose the time-series into trend, cyclical, and seasonal components are discussed. In addition to more informal techniques, some recently developed formal tests for first-order normal and seasonal unit roots are applied. The next two sections contain the empirical results. The data underlying the empirical analysis come from the post-war period and include five industrialized countries, namely the United States, Switzerland, Great Britain, France, and West Germany. The various time-series are measured monthly and quarterly and therefore allow the analysis of seasonal movements. Estimates of the international relationships between real activity variables are presented next. The implications of our findings for business cycle measurement are outlined in the final section of this part.

### 1. Statistical Frameworks

It is common practice in theoretical and empirical work on business cycles to correct the time-series under investigation for growth and seasonal components in order to isolate the cyclical part. The underlying unobserved components model is of the form

$$z_t = T_t + C_t + S_t \quad (1)$$

where  $z$  is the variable under investigation.  $T$ ,  $C$ , and  $S$  are respectively its trend, cyclical, and seasonal parts. Some authors prefer a multiplicative version of equation (1). Empirically, however, we mainly work with the logarithms of data, which may justify an additive formulation.

Two approaches are used to account for secular and seasonal movements in empirical studies of the business cycle. With the first approach, the observable variables, which are considered relevant for economic growth, are explicitly included in the respective regressions. Nelson/Plosser (1982, pg. 153) however note that "using observable variables to account for growth components seems unsatisfactory, since neither factor inputs nor population seem to suffice and direct measures of technology are not readily available."

The second strategy relies on the stochastic properties of the series

itself. The emerging signal extraction problem requires some assumptions about the unobserved individual parts of the series. In the final section of this part, our basic restriction will be that the cyclical component is stationary. But the first task is to identify the class of stochastic processes to which the variable investigated actually belongs. This question is amenable to empirical testing. In accordance with the literature we consider two models. The first model contains a deterministic linear time trend and deterministic seasonal effects plus a stationary stochastic component with an unconditional mean of zero. It is given by the following equation:

$$z_t = \beta_0 + \beta_1 t + \sum_{j=1}^{s-1} \gamma_j D_{jt} + \mu_t \quad (2)$$

where  $t$  denotes a time-trend, and the  $D_j$ 's form an appropriate set of seasonal dummy variables. Note that, in addition to the constant term,  $s-1$  dummies are included.  $s$  is the seasonal periodicity, that is 4 for quarterly and 12 for monthly data.  $\mu$  symbolizes an error term with an unconditional mean of zero but an unspecified autocorrelation structure. The time-series of the estimated  $\mu$ 's is often taken as an adequate measure of the cyclical component in the series  $z_t$ . This procedure is rather obvious from equation (2) because the constant and the trend term proxy for  $T_t$  and the seasonal dummies for  $S_t$ . It is important to note that the secular and seasonal parts of  $z$  are of a deterministic nature, and only the cyclical component is stochastic. We know of no compelling reason to treat the three terms in such an asymmetric way.

A more recently developed alternative to equation (2) is the multiplicative seasonal process proposed by Box/Jenkins (1976). A normal and a seasonal unit root are included to allow for non-stationarities, leading to

$$\phi(L)\Gamma(L^S)(1-L)(1-L^S)z_t = \eta_t \quad (3)$$

$\phi(L)$  and  $\Gamma(L^S)$  are respectively normal and seasonal polynomials in the lag operator  $L$ , defined as  $L^k z_t = z_{t-k}$ . Both  $\phi(L)$  and  $\Gamma(L^S)$  may be of infinite order.<sup>3</sup>  $\eta$  is a white noise random shock. The differenced variable  $w_t =$

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<sup>3</sup>This would be the case if the process (3) contains moving average terms.

$(1-L)(1-L^S)z_t = [(z_t - z_{t-1}) - (z_{t-s} - z_{t-s-1})]$  is assumed to follow a stationary and invertible ARMA-process. In this case, it is not immediately obvious what the cyclical part of  $z$  should be. It will however be possible to treat all parts of  $z$  in a symmetric way, namely as stochastic processes. This topic will be dealt with further in the last section of this part.

In the next two sections, evidence will be presented which yields information about the empirical relevance of the two competing models. First, the autocorrelation structures of  $z_t$  implied by equations (2) and (3) are exploited in this respect. Second, more formal tests for unit roots in the autoregressive representation of a time-series are carried out.

## 2. Autocorrelation Structures

As shown by Chan/Hayya/Ord (1977) and Nelson/Kang (1981, 1984), a first, rather informal test to distinguish between deterministic and stochastic trends can be obtained by comparing the autocorrelation structures of appropriately differenced  $z$ 's and of the estimated residuals from the partly deterministic model (2). A simple case is used to illustrate the relevant implications. Assume model (2) to be correct and the residual  $\mu_t$  to be white-noise. Its estimated counterpart in large samples would have this property too, if the correct detrending and deseasonalizing procedure is used. However if  $z_t$  is inappropriately first and seasonally differenced, the resulting  $w$ 's follow a non-invertible moving-average process and are characterized by zero autocorrelations at all lags except for lags 1 and  $s$ , where they take a value of  $-0.5$ , and lags  $s-1$  and  $s+1$ , with a value of  $0.25$ . If, on the other hand, the stochastic trend model (3) is an adequate representation of the data and the stationary differences  $w$  are white-noise, a regression of  $z$  on a linear trend and seasonal dummies would not produce white-noise residuals, but a spurious estimated autocorrelation function, starting from a value of almost one and dying off only very slowly.

Issues become more complicated if the residuals in equation (2), the  $\mu$ 's, or the stationary differences in equation (3), the  $w$ 's, are respectively serially correlated. Given the results presented below, the case where the residuals in the deterministic trend and seasonal model (2) are highly autocorrelated is especially interesting. Suppose  $\mu_t$  follows the multiplicative ARIMA-process

$$(1-\phi_1L)(1-\Gamma_1L^S)\mu_t = \theta(L)\Delta(L^S)e_t \quad (4)$$

with  $e_t$  white noise.  $w_t$  would then be

$$w_t = \frac{(1-L)(1-L^S)\theta(L)\Delta(L^S)}{(1-\phi_1L)(1-\tau_1L^S)} e_t \quad (5)$$

With  $\phi_1$  and  $\tau_1$  sufficiently close to one,  $w_t$  would be indistinguishable from a pure moving-average process. If  $\theta(L)$  and  $\Delta(L^S)$  are, in addition, of low order,  $w_t$  would show only little serial dependence. As Nelson/Plosser (1982, pg. 147-149) point out, it becomes almost impossible under these circumstances to distinguish in finite samples between deterministic and stochastic trend and seasonal effects on the basis of the autocorrelation properties of the residuals in equation (2) and of the stationary differences in equation (3).

In the empirical work, relevant data for five countries - the United States, Switzerland, Great Britain, France, and West Germany - are used. The emphasis lies on quantity variables representing the state of the real economy, such as real GNP, industrial production, and the unemployment rate. In addition, price variables like real wage rates and real exchange rates are also included. With the exception of unemployment rates, all series are transformed to natural logarithms. The observation period is post World War II and the periodicity is either quarterly or monthly, except for Swiss real GNP, which is available only annually. The data are seasonally unadjusted. Exceptions in that respect are real GNP, industrial production, and the unemployment rate for the United States. The data are from official sources throughout and are available from the authors upon request.

The results, presented in Table 1, are divided into two parts. Regressing the level of the respective series on a time trend and seasonal dummies - shown under the heading "Deterministic Trends" - yields one typical outcome: A very low Durbin-Watson statistic indicating high first-order serial correlation in the estimated residuals. A more complete analysis reveals for all series that the residual autocorrelations start at about 0.9 and decay only very slowly with increasing lag, which is consistent with the random walk hypothesis for the original variables. Note further that the time trend is generally positive and the explanatory power apparently excellent in most cases. Judged according to the significance of the seasonal dummies, about half of the variables contain seasonal elements. Inspection of the serial correlation in the estimated residuals,

however, shows that almost all series still exhibit significant dependencies at seasonal lags.<sup>4</sup> This finding indicates that seasonal dummies alone are not able to capture seasonal effects correctly.

The main characteristics of the autocorrelation function for the various time-series themselves are summarized in the second part of Table 1. Based on the informal identification procedure proposed by Box/Jenkins (1976), all variables must be first and partly seasonally differenced in order to get stationary. The remaining significant autocorrelations generally occur at the first few lags and/or the first seasonal lag, as judged by a two-standard error criterion. Higher order coefficients appear at irregular lags and show no consistent patterns. The variables for which subperiods are examined furthermore reveal that the serial dependence seems to be quite unstable over time.<sup>5</sup> The results for real exchange rates observed over the recent flexible rate period are especially interesting, because the random walk provides an appropriate model as in the case of nominal rates.

The general conclusion seems to be that all series are generated by low order moving average processes. Remember, however, as noted above, that these results are also consistent with the hypothesis that the cyclical component, measured as the deviation from a deterministic trend, is highly autocorrelated. Our results are furthermore consistent with the findings of Nelson/Plosser (1982) for U.S. data measured annually. They observe positive serial correlation at lag one in first differences, which is, however, most likely due to the averaging of shorter interval observations.

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<sup>4</sup>The autocorrelation function of the first differenced residuals proved to be very useful for that purpose.

<sup>5</sup>The subperiods have been formed according to the graphical picture of the series.

TABLE 1

## Statistical Characteristics

Variable	Obs. Per.	N	Deterministic Trends				Time-Series Analysis	
			Trend	Seas. Dummies	R <sup>2</sup>	DW	Stat. Diff.	Sign. Autocorr.
<u>United States</u>								
Real GNP	1/47-1V/81	140	+	0	0.99	0.12	1.0	1:0.35, 2:0.21
Industrial Prod.	1/47-2/82	422	+	0	0.98	0.04	1.0	1:0.48, 2:0.29, 3:0.18, 12: -0.22, 13: -0.20, 14: -0.23, 15: -0.16
Unemployment Rate	1/48-2/82	410	+	0	0.27	0.04	1.0	1:0.16, 2:0.27, 3:0.19, 4:0.15, 5:0.17, 10:-0.14, 12:-0.22, 15:-0.12, 24:-0.13
	2/48-1/63	180	+	0	0.16	0.07	1.0	2:0.32, 3:0.21, 5:0.24, 10:-0.20, 12:-0.27
	2/63-2/70	85	-	0	0.80	0.18	1.0	3:-0.24, 4:0.35
	3/70-2/82	144	+	0	0.22	0.05	1.0	1:0.22, 2:0.34, 3:0.17, 4:0.26
Real Wage	1/64-2/82	218	+	0	0.17	0.01	1.1	3:0.21, 9:0.14, 11:0.16, 12:-0.22, 24:-0.29
	5/64-8/72	100	+	0	0.96	0.18	1.1	12:-0.31
	9/72-2/82	114	-	0	0.56	0.03	1.1	2:0.24, 3:0.30, 24:-0.24
<u>Switzerland</u>								
Real GNP	1948-1981	34	+		0.95	0.18	1	
Industrial Prod.	1/61-1/83	89	+	✓	0.75	0.17	1.1	4:-0.42
Employment	1/61-1/83	89	-	✓	0.21	0.21	1.1	
<u>France</u>								
Real GNP	1/70-1/83	53	+	0	0.95	0.10	1.0	2:0.35
Industrial Prod.	1/70-6/83	162	+	✓	0.91	0.40	1.1	1:0.42, 12:0.27
Real Wage	1/68-11/83	62	+	0	0.97	0.15	1.0	
<u>Great Britain</u>								
Real GNP	1/60-1/83	93	+	✓	0.92	0.29	1.1	4:-0.46, 6:0.38, 10:-0.34
Industrial Prod.	1/60-1/83	93	+	✓	0.76	0.14	1.1	3:0.25, 4:-0.49
Unemployment Rate	1/56-6/83	330	+	0	0.66	0.01	1.1	1:0.35, 2:0.15, 3:0.14, 4:0.18, 12:-0.32
	1/56-12/70	180	+	✓	0.31	0.09	1.1	1:0.34, 12:-0.44 13:-0.28
	1/76-12/79	48	0	0	0.30	0.12	1.1	
Real Wage	1/63-5/83	245	+	0	0.92	0.15	1.1	1:-0.22, 12:-0.42 13:0.24
	1/63-12/72	120	+	0	0.96	0.47	1.1	1:-0.49, 2:0.22, 12:-0.49 13:0.41,
	1/73-12/82	120	+	0	0.51	0.22	1.1	12:-0.39

Table 1 Continued

## Statistical Characteristics

Variable	Obs. Per.	N	Deterministic Trends				Time-Series Analysis		
			Trend	Seas. Dummies	$\bar{R}^2$	DW	Stat. Diff.	Sign. Autocorr.	
<u>West Germany</u>									
Real GNP	1/60-1V/82	92	+	✓	0.96	0.42	1.1		
Industrial Prod.	1/63-12/82	240	+	✓	0.85	0.37	1.1	1:-0.60, 3:0.19, 4:-0.21, 7:-0.14, 11:0.20, 12:-0.26, 23:0.27, 24:-0.21	
Unemployment Rate	1/68-1/83	181	+	✓	0.77	0.03	1.1	1:0.35, 2:0.31, 3:0.35, 4:0.22, 7:0.16, 12:-0.20	
	1/68-8/74	80	+	✓	0.36	0.09	1.1		
	1/75-8/81	80	-	✓	0.45	0.08	1.1	1:0.35, 2:0.24, 3:0.37, 12:-0.28	
<u>Real Exchange Rates</u>									
S.Fr./\$	1/73-2/82	110	-		0.25	0.12	1.0		
S.Fr./£	1/73-6/83	126	+		0.11	0.09	1.0		
S.Fr./FF	1/73-6/83	126	-		0.46	0.15	1.0		
S.Fr./DM	1/73-2/83	121	-		0.71	0.16	1.0	1:0.19	

## Notes:

- Obs. Per.: Observation period  
N: Number of observations  
0: Not significantly different from zero  
+, -: Significantly positive, negative  
✓: Significantly different from zero  
 $\bar{R}^2$ :  $R^2$ , adjusted for degrees of freedom  
DW: Durbin-Watson statistic  
Stat. Diff.: Stationary differences. Given is the order of normal differencing, followed by the order of seasonal differencing.  
Sign. Autocorr.: Autocorrelations significantly different from zero, judged by a two-standard error criterion. Given is the lag, followed by the estimated serial correlation coefficient.

### 3. Unit Root Tests

More formal test procedures, relevant to the models investigated in this study, have recently been developed by Fuller (1976) and Hasza/Fuller (1982a,b).<sup>6</sup> The starting point for these procedures is a regression equation combining the two models (2) and (3), assuming purely autoregressive representations of finite order. The following discussion distinguishes between time-series containing seasonal elements and series which do not. The distinction is drawn according to the results presented above, that is, based on the significance of seasonal dummies and autocorrelation coefficients at seasonal lags respectively.

The model for time-series with no seasonal characteristics is taken from Fuller (1976) and given by

$$z_t = \alpha_1 z_{t-1} + g_1 v_{t-1} + \dots + g_p v_{t-p} + \beta_0 + \beta_1 t + \epsilon_t \quad (6)$$

where  $v_t = z_t - z_{t-1}$ . The relevant possible unit root is isolated through the coefficient  $\alpha_1$ . The hypothesis  $\alpha_1 = 1$  is tested by comparing the usual regression t-value to  $\hat{\tau}_\tau$  in Fuller (1976, Table 8.5.2). All series, irrespective of seasonality, are examined. Therefore, possible seasonal characteristics are neglected, except for the number  $p$  of lagged differences included in the regression. Note that  $p$  extends to the first seasonal lag in all cases.

The results based on estimating equation (6) by ordinary least squares are presented in Table 2. For almost all series, the hypothesis of a first order unit root cannot be rejected. The trend parameter  $\beta_1$  moreover becomes insignificant for the majority of the variables as judged by the usual t-statistic. The  $\bar{R}^2$  and the Durbin-Watson statistic indicate no further problems. Only the West German unemployment rate deviates considerably from this outcome. These findings are consistent with the estimates reported by Nelson/Plosser (1982).

For time-series containing seasonal elements, the most general model is chosen from Hasza/Fuller (1982b). It is given by

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<sup>6</sup>In addition to the literature, private correspondence with Professor Fuller was very helpful in carrying out the tests.

$$\begin{aligned}
z_t = & \alpha_1 z_{t-1} + \alpha_2 (z_{t-s} - z_{t-s-1}) + \alpha_3 (z_{t-1} - z_{t-s-1}) \\
& + k_1 w_{t-1} + \dots + k_h w_{t-h} \\
& + \beta_0 + \beta_1 t + \sum_{j=1}^{s-1} \gamma_j D_{jt} + \epsilon_t
\end{aligned} \tag{7}$$

with  $w_t = (1-L)(1-L^S)z_t$ .  $h$  is equal to  $p+sP$  where  $p$  and  $P$  denote respectively the order of the normal and seasonal autoregressive part. The unit roots are again isolated as coefficients.  $\alpha_1$  and  $\alpha_2$  are equal to one and  $\alpha_3$  equal to zero, if a first-order normal and a first-order seasonal unit root are present. Under these circumstances, equation (7) can be written in the normal and seasonal differences as

$$\phi(L)\Gamma(L^S)(1-L)(1-L^S)z_t = \beta_0 + \beta_1 t + \sum_{j=1}^{s-1} \gamma_j D_{jt} + \epsilon_t \tag{8}$$

The following hypotheses are tested by estimating equation (7) with ordinary least squares:

H1:  $\alpha_1 = 1$ , assuming that the seasonal model is stationary. The test-statistic is calculated as a conventional t-value relative to 1.0.  $\hat{\tau}_\tau$  from Fuller (1976, table 8.5.2) is used to perform the test.

H2:  $\alpha_1 = \alpha_2 = 1$  and  $\alpha_3 = 0$ . A conventional F-statistic is calculated and compared to the relevant value of  $\phi_{n-d-4}^{(3)}$  in Hasza/-Fuller (1982, table 5.1).

H3:  $\alpha_1 = \alpha_2 = 1$  and  $\alpha_3 = \beta_0 = \beta_1 = \gamma_j$  (all  $j$ ) = 0. Again, the usual F-value is taken. In this case,  $\phi_{n-d-4}^{(d+4)}$  is the relevant entry in Hasza/Fuller (1982, table 5.1).

The findings based on equation (7) are shown in Table 3. The hypothesis H1,  $\alpha_1 = 1$ , cannot be rejected. The seasonal model is assumed to be

TABLE 2

## Tests for First-Order Unit Roots

$$\text{Estimated equation: } z_t = \alpha_1 z_{t-1} + g_1 v_{t-1} + \dots + g_p v_{t-p} + \beta_0 + \beta_1 \dagger + \varepsilon_t$$

Variable	Obs. Per.	N	p	$\alpha_1$	$\beta_0$	$\beta_1$	$\bar{R}^2$	DW
<u>United States</u>								
Real GNP	1/47-1V/81	140	8	0.92 (-2.33)	0.53 (2.38)	0.0007 (2.21)	0.99	2.02
Industrial Prod.	1/47-2/82	422	12	0.98 (-2.11)	0.08 (2.20)	0.00007 (1.99)	0.99	1.98
Unemployment Rate	1/48-2/82	410	12	0.97 (-2.60)	0.10 (2.05)	0.0002 (2.01)	0.98	2.00
	2/48-1/63	180	12	0.94 (-2.46)	0.20 (1.91)	0.0009 (1.64)	0.96	2.01
	2/63-2/70	85	12	0.98 (-0.25)	-0.10 (-0.12)	0.0007 (0.31)	0.97	2.02
	3/70-2/82	144	12	0.95 (-2.30)	0.00 (-0.01)	0.0009 (1.64)	0.97	1.98
Real Wage	1/64-2/82	218	12	0.98 (-2.51)	0.02 (2.70)	0.000003 (0.38)	0.99	1.97
	5/64-8/72	100	12	0.75 (-2.83)	0.16 (2.74)	0.0003 (2.98)	0.99	2.11
	9/72-2/82	114	12	0.92* (-3.73)	0.10 (3.56)	-0.00005 (-2.38)	0.98	2.06
<u>Great Britain</u>								
Real GNP	1/60-1/83	93	8	0.87 (-1.92)	1.18 (1.96)	0.0005 (1.32)	0.98	2.01
Industrial Prod.	1/60-1/83	93	8	0.89 (-1.90)	0.49 (1.96)	0.0003 (0.97)	0.95	1.84
Unemployment Rate	1/56-6/83	330	12	0.98 (-3.28)	-0.02 (-0.93)	0.0007 (3.35)	0.99	1.77
	1/56-12/70	180	12	0.88* (-3.63)	0.17 (3.10)	0.0006 (2.15)	0.92	1.87
	1/76-12/79	48	12	0.80 (-2.50)	2.54 (1.54)	-0.005 (-0.99)	0.86	1.90
Real Wage	1/63-5/83	245	12	0.95 (-1.71)	0.22 (1.76)	0.00008 (1.47)	0.99	2.03
	1/63-12/72	120	12	0.89 (-0.97)	0.48 (0.97)	0.0003 (1.15)	0.98	1.99
	1/73-12/82	120	12	0.84 (-2.36)	0.73 (2.36)	0.0001 (1.98)	0.87	2.03

TABLE 2 Continued  
Tests for First-Order Unit Roots

Variable	Obs. Per.	N	p	$\alpha_1$	$\beta_0$	$\beta_1$	$\bar{R}^2$	DW
<u>Switzerland</u>								
Real GNP	1948-1981	34	5	0.95 (-0.30)	0.56 (0.36)	0.0001 (0.02)	0.99	2.02
Industrial Prod.	1/61-1/83	89	8	0.91 (-1.59)	0.47 (1.68)	0.0002 (0.53)	0.93	1.86
Employment	1/61-1/83	89	8	0.92 (-2.38)	0.38 (2.39)	-0.00009 (-1.94)	0.96	1.95
<u>France</u>								
Real GNP	1/70-1/83	53	8	0.91 (-1.07)	1.10 (1.09)	0.0004 (0.60)	0.99	2.01
Industrial Prod.	1/70-6/83	162	12	0.85 (-1.97)	0.69 (2.05)	0.00002 (0.26)	0.96	2.58
Real Wage	1/68-11/83	62	8	0.94 (-1.05)	0.27 (1.14)	0.0003 (0.60)	0.99	2.04
<u>West Germany</u>								
Real GNP	1/60-IV/82	92	8	0.96 (-0.58)	0.21 (0.64)	0.0001 (0.25)	0.99	2.00
Industrial Prod.	1/63-12/82	240	12	0.97 (-0.58)	0.16 (0.69)	-0.00004 (-0.33)	0.95	2.21
Unemployment Rate	1/68-1/83	181	12	2.01** (5.91)	0.28 (0.87)	0.008 (2.86)	0.88	0.14
	1/68-8/74	80	12	1.17 (1.27)	1.07 (3.20)	0.0008 (0.25)	0.87	0.82
	1/75-8/81	80	12	3.93** (18.38)	-1.82 (-6.00)	0.002 (2.88)	0.95	1.75
<u>Real Exchange Rates</u>								
S.Fr./\$	1/73-2/82	110	12	0.94 (-1.42)	0.24 (0.90)	0.00005 (0.18)	0.91	1.99
S.Fr./£	1/73-6/83	126	12	0.94 (-2.30)	0.22 (1.96)	0.0001 (1.49)	0.93	1.98
S.Fr./FF	1/73-6/83	126	12	0.89 (-2.57)	0.50 (2.52)	-0.0002 (-1.75)	0.92	2.00
S.Fr./DM	1/73-2/83	121	12	0.84 (-3.12)	0.78 (3.11)	-0.0004 (-2.80)	0.96	1.98

Notes:

$$v_t = z_t - z_{t-1}$$

Obs. Per.: Observation period

N: Number of observations

p: Number of lagged differences included. The degrees of freedom of the estimated regressions are  $N-p-3$ .

$\bar{R}^2$ :  $R^2$ , adjusted for degrees of freedom

DW: Durbin-Watson statistic

t-values are shown in parentheses below the estimated coefficients. The t-value for  $\alpha_1$  is given relative to 1.0.

\*  $\alpha_1$  is significantly different from one at the 5%-level

\*\*  $\alpha_1$  is significantly different from one at the 1%-level

TABLE 3

Tests for First-Order and First-Order Seasonal Unit Roots

Estimated equation (full model):

$$z_t = \alpha_1 z_{t-1} + \alpha_2 (z_{t-2} z_{t-s-1}) + \alpha_3 (z_{t-1} z_{t-s-1}) + k_1 w_{t-1} + \dots + k_h w_{t-h} + \beta_0 + \beta_1 t + \sum \gamma_j D_{jt} + \varepsilon_t$$

Variable	Obs. Per.	N	h	FM(SD)			FM			Unit Roots Tests		Seasonal Dummies	
				$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\phi_{n-d-4}^{(3)}$	$\phi_{n-d-4}^{(d+4)}$	FM(SD)	RM
<u>United States</u>													
Real Wage	1/64-2/82	218	24	0.99 (-0.11)	0.07	-0.05	1.00 (0.09)	0.81	-0.12	16.18*	3.29	2.63**	0.24
	5/64-8/72	100	24	-0.28 (-3.11)	-0.77	1.45	0.62 (-2.28)	0.70	0.02	6.68	1.78	1.41	0.39
	9/72-2/82	114	24	0.98 (-0.48)	-0.18	-0.02	0.99 (-0.31)	0.81	-0.11	13.70*	2.93	2.56**	0.29
<u>Switzerland</u>													
Industrial Prod.	1/61-1/83	89	8	0.93 (-1.15)	0.55	-0.22	0.93 (-1.12)	0.95	-0.33	5.42	2.18	2.49	0.25
Employment	1/61-1/83	89	8	0.93 (-1.77)	0.76	-0.04	0.94 (-1.54)	0.90	-0.09	6.67	2.61	2.25	0.10
<u>France</u>													
Industrial Prod.	1/70-6/83	162	24	0.59 (-3.05)	0.07	0.36	1.01 (0.64)	1.03	-0.37	15.49*	3.60*	4.04	0.64
<u>West Germany</u>													
Real GNP	1/60-11/82	92	8	0.94 (-1.00)	0.82	-0.29	0.98 (-0.42)	0.91	-0.43	5.48	3.47	1.43	2.53
Industrial Prod.	1/63-12/82	240	24	0.95 (-0.96)	0.32	-0.20	0.96 (-0.88)	0.93	-0.33	17.62**	3.58	3.46*	0.30

Unemployment Rate	1/68-1/83	181	24	0.94 (-2.38)	0.57	0.10	0.96 (-2.37)	1.02	0.03	6.76	1.92	1.60	0.71
	1/75-8/81	80	24	0.96 (-1.08)	0.13	0.12	0.98 (-0.86)	1.00	0.03	10.21	4.38*	5.00**	1.44
<u>Great Britain</u>													
Real GNP	1/60-1/83	93	8	0.90 (-1.32)	0.74	-0.67	0.96 (-0.62)	0.93	-0.85	10.19*	4.30*	2.10	0.90
Industrial Prod.	1/60-1/83	93	8	0.92 (-1.21)	0.44	-0.31	0.92 (-1.34)	0.91	-0.40	18.22**	4.31*	3.02	1.16
Unemployment Rate	1/56-6/83	330	24	1.02 (1.67)	0.56	-0.12	1.02 (-1.47)	0.84	-0.13	16.01*	3.31	2.10*	0.34
	1/56-12/70	180	24	0.91 (-1.33)	-0.38	0.08	0.92 (-1.59)	0.72	-0.02	25.36**	5.02**	4.40**	0.17
Real Wage	1/63-5/83	245	24	0.95 (-1.52)	0.36	-0.01	0.95 (-1.42)	0.68	-0.04	13.41	2.84	1.78	0.37
	1/63-12/72	120	24	0.83 (-1.01)	0.26	-0.08	0.98 (-0.21)	0.79	-0.45	6.50	2.02	1.38	0.70
	1/70-12/82	120	24	0.78 (-2.31)	-0.26	0.22	0.84 (-1.64)	0.62	0.07	19.05**	4.03*	4.09**	0.36

Notes:

$w_t = (z_t - z_{t-1}) - (z_{t-s} - z_{t-s-1})$

Obs. Per.: Observation period

N: Number of observations

h: Number of lagged differences included

FM(SD): Full model with seasonal dummies

FM: Full model without seasonal dummies

RM: Restricted model ( $\alpha_1 = \alpha_2 = 1, \alpha_3 = 0$ ) with seasonal dummies

$\phi_{n-d-4}^{(3)}$ : Test for  $\alpha_1 = \alpha_2 = 1$  and  $\alpha_3 = 0$

$\phi_{n-d-4}^{(d+4)}$ : Test for  $\alpha_1 = \alpha_2 = 1$  and  $\alpha_3 = \beta_0 = \beta_1 = \gamma_j$  (all  $j$ ) = 0

The degrees of freedom of the estimated regressions are at least  $N-h-s-4$ .

The t-value of  $\alpha_1$ , in parentheses, is given relative to 1.0.

\* The tested hypothesis is rejected at the 5% level.

\*\* The tested hypothesis is rejected at the 1% level.

stationary in this case. A comparison of the columns FM(SD) and FM demonstrates, moreover, that the outcome does not depend on the inclusion or exclusion of seasonal dummies in equation (7). The evidence on the joint hypothesis H2, that a first-order and a first-order seasonal unit root are present, is mixed. Out of 17 series examined,  $\phi_{n-d-4}^{(3)}$  is below the critical value in 8 cases. The values for  $\phi_{n-d-4}^{(d+4)}$  show that this number increases to 11 if H3 is tested, that is, if the parameters on the time-trend and the seasonal dummies are set equal to zero under the null hypothesis. Two additional characteristics must be noted. First, the seasonal dummies lose their significance completely if both  $\alpha_1$  and  $\alpha_2$  are restricted to one and  $\alpha_3$  to zero (last column in Table 3). It appears that a seasonal time series model including one unit root describes seasonality quite well. As mentioned above, this is not the case for seasonal dummies alone. Second, a comparison of FM(SD) and FM reveals that the value of  $\alpha_2$  increases considerably towards one if seasonal dummies are excluded from the model.

One possible caveat must be mentioned at this point. The test procedures used above have virtually no power in discriminating between a first-order unit root and an autocorrelation coefficient slightly below one. However, the economic interpretation is completely different in the two cases. If a unit root is present, the series is non-stationary, whereas in the other case, it would return to a constant long-run mean and would therefore exhibit long-run cyclical swings. Three characteristics of the empirical results favor the unit root interpretation. First, the parameter  $\alpha_1$  is close to one irrespective of the measurement interval. However, given the length of a possible cyclical adjustment period,  $\alpha_1$  should be inversely related to the time interval between observations. Second, the time required to return to the long-run mean is implausibly long on economic grounds if the  $\alpha_1$  coefficient is actually around .9. Third, the uniformity of the results across countries and time-series is also in favor of the unit root interpretation, unless one accepts the proposition that the speed of adjustment is virtually the same in all markets and countries.

The results presented in this and the preceding sections are easily summarized. In no case can the hypothesis of a first-order unit root be rejected. This means that it is highly likely that a stochastic trend or random walk component is present. First differencing of the data is therefore required to get a stationary time-series. Seasonal patterns, if they are present, are better captured through a seasonal ARIMA-model than

through seasonal dummy variables. Generally, a seasonal unit root is involved, meaning that seasonal differencing of the data is an adequate empirical procedure. The remaining stationary part of the series generally follows a pure moving average process of relatively low order. Based on these findings, an adequate statistical representation of all variables considered would be a variant of

$$(1-L)(1-L^S)z_t = \theta_q(L)\Delta_Q(L^S)\eta_t \quad (9)$$

where  $q$  and  $Q$  denote the orders of the  $\theta$ - and  $\Delta$ - polynomials respectively.<sup>7</sup> For variables containing seasonality,  $Q$  would generally equal 1, and  $q$  would be smaller than  $s$  with possibly some intermediate parameters restricted to zero. For non-seasonal series  $(1-L^S)$  and  $\Delta(L^S)$  would both be one.

#### 4. International Connections

The international connections between real magnitudes are investigated with various econometric techniques and for two different time periods. The quarterly series for real GNP and industrial production are alternatively used for that purpose. The statistical work includes the estimation of ARIMA-models as well as bivariate cross-correlograms and distributed lag regressions for the stationary differences and the ARIMA-residuals. The results allow some general conclusions. First, all series examined can be well described by an ARIMA-process in the normal and seasonal differences including one seasonal moving-average parameter. Second, the bivariate cross-correlations indicate positive feedback relationships among most countries. Not unexpectedly, the influences from the United States on the West European countries seem to be stronger than vice versa. Third, the lags are surprisingly short. The strongest effects even occur contemporaneously in most cases. A representative example of the empirical results is shown in Table 4. The regressions are estimated by ordinary least squares in the stationary, that is first and seasonal, differences of the industrial production indices over the period 1965-1981. A constant term

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<sup>7</sup>Plosser (1979) observes that the endogenous variables of a dynamic simultaneous equation model do not necessarily follow stochastic processes where the moving-average portion factors into the product of seasonal and non-seasonal polynomials. This issue is not pursued further here.

TABLE 4

## International Relationships - An Example

Exogenous: Industrial Production	Endogenous: Industrial Production			
	United States	Switzerland	Great Britain	West Germany
<u>United States</u>				
Lag: 0		0,33 (1,89)		0,41 (4,10)
4		0,29 (1,52)		
<u>Switzerland</u>				
Lag: 0				0,19 (3,02)
<u>Great Britain</u>				
Lag: 0				0,27 (2,80)
3		0,32 (2,16)		
<u>West Germany</u>				
Lag: 0	0,51 (5,41)	0,48 (2,69)	0,38 (4,35)	
4		0,41 (2,28)		
$\bar{R}^2$	0,55	0,50	0,46	0,53
DW	1,62	2,05	2,06	2,00

## Notes:

The regressions are estimated in the first and seasonal differences of the natural logarithms of the variables. Four lagged dependent variables are included. t-values are shown in parentheses below the estimated coefficients.

$\bar{R}^2$ :  $R^2$ , adjusted for degrees of freedom

DW: Durbin-Watson statistic

and four lagged dependent variables are also included. It is easily seen that significantly positive effects are observed with only a small or no time lag.

These findings suggest that the real economies of industrialized countries are fairly well integrated. The question concerning the dominant impulse forces, however, receives no answer from the evidence presented here. One possibility would be that policy actions or real shocks in a large country, for example the United States, are transmitted to the rest of the world via the demand for import goods. An alternative explanation would stress the common reaction of all countries to worldwide influences, such as the oil price increases over the last decade.

### 5. Implications for Business Cycle Measurement

The model given by equation (9), found to be an adequate representation of the time-series under investigation, is now explored with respect to the qualitative characteristics of the implied cyclical component, which is assumed to be stationary with an unconditional mean of zero. The autocorrelation structure of the cyclical measure, describing the persistence of the business cycle, is of primary importance in that context. In order to allow the maximum possible degree of persistence, the trend component is limited to a random walk with drift, whereas all seasonal regularities are attributed to the seasonal part. The procedures followed are equivalent to the ones used by Beveridge/Nelson (1981) and Nelson/Plosser (1982), who, however, consider exclusively non-seasonal models. Based on equations (1) and (9) and taking  $Q = 1$ , the model then becomes

$$\begin{aligned} (1-L)T_t &= \eta_{1t} + K \\ (1-L^S)S_t &= (1-\Delta_1 L^S) \eta_{2t} \end{aligned} \tag{10}$$

$$C_t = f(L) \eta_{3t}$$

and therefore

$$\begin{aligned} (1-L)(1-L^S)z_t = w_t &= (1-L^S) \eta_{1t} + (1-L)(1-\Delta_1 L^S)\eta_{2t} \\ &+ (1-L)(1-L^S)f(L)\eta_{3t} \end{aligned} \tag{11}$$

$\eta_{1t}$ ,  $\eta_{2t}$  and  $\eta_{3t}$  are mutually uncorrelated white noise sequences and  $K$  is a constant term, which vanishes in equation (11) because seasonal differ-

encing is applied to it. It becomes already obvious at this point that real exchange rates, since they follow random walks, must be governed exclusively by a stochastic trend component.

The autocorrelation structure of  $w$  implied by equation (11) is finite if the order of  $f(L)$  is finite. The first component on the right hand side, associated with  $\eta_{1t}$ , generates serial dependencies at the seasonal lag only. Autocorrelations different from zero at lags 1,  $s-1$ ,  $s$  and  $s+1$  are given through  $\eta_{2t}$ . The last part yields autocorrelations from lag 1 up to lag  $F+s+1$  with intermediate zeros at lags  $F+2$  to  $s-F-2$ , where  $F$  is the order of  $f(L)$ . For quarterly (monthly) data, a moving average process for the cyclical component of order 1 (5) would therefore imply an uninterrupted, but most likely irregular series of non-zero serial correlation coefficients for the stationary differences in the variable under investigation. These features correspond quite closely to the empirical findings presented above. Given the observed time-series structures,  $f(L)$  must therefore obviously be of low order. Even if there exists no cyclical component, that is if  $f(L) = 0$ , the autocorrelation coefficients at lags 1,  $s-1$  ( $=s+1$ ) and  $s$  are non-zero and take the values  $-(1+\Delta_1^2) \sigma_2^2/\gamma_0$ ,  $\Delta_1 \sigma_2^2/\gamma_0$  and  $-(\sigma_1^2 + 2\Delta_1 \sigma_2^2)/\gamma_0$ , where  $\gamma_0 = 2\sigma_1^2 + 2(1+\Delta_1^2)\sigma_2^2$  is the variance of  $w_t$ .  $\sigma_1^2$  and  $\sigma_2^2$  denote the variances of  $\eta_1$  and  $\eta_2$  respectively. If the cyclical component is pure white noise, that is equal to  $\eta_{3t}$ , the correlogram of  $w$  extends to  $s+1$ , with intermediate zeros at lags 2 to  $s-2$ .

Based on the empirical results and the analysis presented in this section, it must be concluded that the cyclical component, if it exists at all, contains very little forecastable momentum.<sup>8</sup> Therefore, it appears that cyclical shocks create virtually no persistence in real magnitudes. This does, of course, not exclude the possibility that "business cycles" are created by a series of independent shocks in the same direction. These conclusions remain unchanged if a business cycle index is considered which

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<sup>8</sup>For an alternative approach which yields similar results, see McCulloch (1975).

is defined as a weighted linear combination of individual series.<sup>9</sup>

The characterization of the cyclical component in macroeconomic time-series presented above is in sharp contrast to the highly autocorrelated business cycle indicators measured as the deviation from a deterministic time-trend and fixed seasonal effects. A representative example, the Swiss index of industrial production, is useful to clarify the point. In Figure 1, the time-series of estimated residuals from a regression of the natural logarithm of industrial production on a trend and seasonal dummies is compared to the stationary differences in the natural logarithm of this index. The differences are rather obvious. The residuals exhibit large swings, whereas the differencing operation results in a highly random series. In the empirical literature on business cycles, the trend component is usually proxied by a deterministic trend. Based on our findings, this approach is misleading. As already mentioned, the high degree of persistence in the resulting business cycle indicator is totally spurious, depending exclusively on sample size. Wasserfallen (1985) furthermore shows that test results for business cycle models may crucially depend on the chosen measurement procedures. This is not surprising because a highly autocorrelated dependent variable is certainly "explained" more easily by the usual distributed lag regressions than an almost purely random series.<sup>10</sup>

### III. MACROECONOMIC POLICIES AND THE DISTRIBUTION OF THE OUTPUT RATE

Macroeconomists traditionally assume that the output rate follows a stochastic process, which exhibits a constant mean and serially correlated deviations from the mean. With this traditional approach, the deviations of the output rate from its mean characterize the business cycle. The empirical evidence presented in this paper suggests that a new view of the distribution of the output rate is at least as plausible as the traditional

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<sup>9</sup>The indicator approach conforms to the classical definition of the business cycle by Burns/Mitchell (1946). They state on page 6: "Our definition presents business cycles as a consensus among expansions in 'many' economic activities followed by 'similarly general' recessions, contractions, and revival."

<sup>10</sup>Nelson/Kang (1984) note a number of important additional pitfalls associated with the use of wrongly detrended variables.

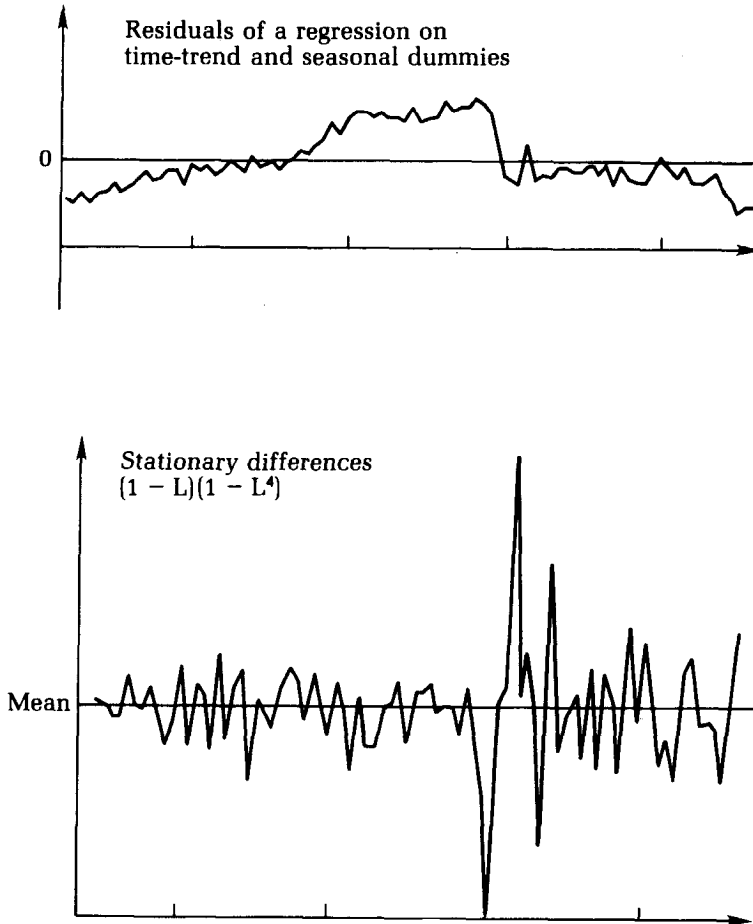


Figure 1

Quarterly index of industrial production  
Switzerland 1961-1983

view. This new view is that the mean output rate changes stochastically over time and that there is little serial correlation in the deviations of the output rate from its mean.

With the traditional view of the distribution of the output rate, growth theory focuses on explaining the mean of the output rate, while macroeconomists try to explain the deviations of the output rate from its mean. The empirical evidence presented in this paper suggests that changes in the mean explain a larger fraction of the fluctuations of the output rate than the deviations from the mean. It follows that with the new view, if one wants to explain the fluctuations of the output rate, one has to explain changes in its mean as well as its deviations from the mean.

In this part of the paper, we build a general equilibrium model which reflects the new view. The distribution of the output rate derived in the model mimics the facts about the empirical distribution of the output rate presented in Part II of this paper. The model shows that the mean and the variance of the output rate can change stochastically, both because of factors associated with growth theory - in this case, real shocks which affect the technologies used to produce output - and factors associated with business cycle theory - in this case, unanticipated changes in the mean and the variance of the growth rate of the money stock.

This part of the paper is organized as follows. Section III.1. provides an informal discussion of the model and of some of the results. Section III.2. presents the optimization problem of households. Section III.3. discusses the solution of that problem. Section III.4. derives the nominal rate of interest. Finally, Section III.5. derives the real rate of interest and the distribution of the output rate.

## **1. An Informal Discussion of the Model**

The model developed in this part of the paper emphasizes the fact that the mean and the variance of the output rate are solved for households. The households choose consumption and investment policies which maximize their lifetime expected utility. Unless there is only one commodity produced with only one technology, the mean and the variance of the output rate depend on how households allocate their resources over technologies and commodities.

The model developed in this part of the paper lets infinitely-lived households choose the distribution of the output rate in an economy with perfect markets. Only one commodity is produced in the economy considered here, and it can be produced by a variety of constant stochastic returns to

scale technologies which change randomly over time. It is assumed that the quantity of the commodity invested in each technology can be changed costlessly at each point in time. The mix of technologies used by households determines the distribution of the output rate. Households consume both the commodity and the services of real balances.

Households are assumed to be risk-averse. In the absence of money, households would choose the mean and the variance of the output rate in the same way as risk-averse investors choose how to invest their wealth among various risky assets. Households would invest in a diversified portfolio of technologies, and if they were sufficiently risk-averse, they would try to hedge against unanticipated changes in the technologies. In this case, if households exhibit constant relative risk tolerance, the mix of technologies used in production would change over time only because technologies change over time. The existence of money can lead households to choose a different mix of technologies than they would choose in the absence of money, as the introduction of money and nominal assets expands the investment opportunity set of households.

In the model developed here, monetary policy plays an important role in the determination of the distribution of the output rate. It is assumed that the government uses its control of the stock of money to try to achieve a target rate of change for the price of money. The target rate of change for the price of money follows a stochastic process to take into account the fact that monetary policy changes unexpectedly over time. However, the government's control of the money stock is assumed to be imperfect, so that the actual growth rate of the money stock is the growth rate required to achieve the government's target growth rate for the price of money plus a random variable. Consequently, the actual growth rate of the price of money is itself a random variable. The deviations of the growth rate of the price of money from its target growth rate are assumed to be serially uncorrelated.

Households can invest in the production of the commodity, in real balances, in default-free nominal bonds, and in default-free bonds which offer a safe real rate of return. There is no outside supply of default-free bonds. Consequently, the nominal rate of interest must be such that the aggregate demand for nominal bonds is equal to zero. In equilibrium, the nominal rate of interest is a decreasing function of the expected rate of change of the price of money. As the opportunity cost of real balances is an increasing function of the nominal rate of interest, it follows that a decrease in the expected rate of change of the price of money (which is

equivalent to an increase in the expected rate of inflation) decreases the households' holdings of real balances. In this model, the households' real wealth is the sum of the stock of the commodity and of the real balances. Consequently, an increase in the nominal rate of interest decreases the households' real wealth.

Throughout the paper, it is assumed that households exhibit constant relative risk aversion and constant expenditure shares. Furthermore, nominal and real assets are assumed to be gross substitutes. With these assumptions about the households' utility function and the joint distribution of real returns, a decrease in the households' real wealth decreases the amount of real wealth they want to invest in a given portfolio of risky investments. Consequently, an increase in the nominal rate of interest decreases the total value of the households' investments in the production of the commodity for a given distribution of real returns on investments in production. The change in the nominal rate of interest does not affect the stock of the commodity. This means that, for a given joint distribution of real returns on investments in production, not all of the stock of the commodity is invested in production following an increase in the nominal rate of interest. Hence, it is necessary that investments in production be made more attractive to the households to re-establish equilibrium. The expected real returns on investments in production are given by the technologies and cannot be altered. However, a decrease in the real rate of interest makes it less attractive for households to invest in bonds and more attractive to invest in production. Therefore, a fall in the real rate of interest must accompany an increase in the nominal rate of interest to maintain equilibrium in the economy. While an increase in the nominal rate of interest also brings about a fall in the real rate of interest in the models of Mundell (1963) and Tobin (1965), this result is derived here in a model in which households maximize explicitly their expected utility of lifetime consumption.

The difference between the expected output rate and the rate of interest corresponds to the risk premium implicitly paid to households for bearing the risk associated with production. A fall in the real rate of interest increases this risk premium for a given distribution of the output rate. This change in the risk premium reflects the fact that households are less willing to bear a given amount of risk following the change in the nominal rate of interest. In this model, however, households cannot decrease the risk they bear by investing more in the safe asset, as there is no outside supply of the safe asset. Consequently, households must change

the distribution of output to decrease the amount of risk they bear. Whenever the risk of invested wealth is an increasing function of the variance of output, as it is in this model, households can reduce the amount of risk they bear by decreasing the variance of the output rate. However, if households chose an efficient mix of technologies before the increase in the nominal rate of interest, the decrease in the variance of the output rate must be accompanied by a decrease in the expected output rate. Consequently, an increase in the nominal rate of interest decreases both the mean and the variance of the output rate. The assumption that households can choose among many technologies is crucial to obtain this result. As in this model an increase in the variance of the growth rate of the money stock increases the nominal rate of interest through a risk premium effect, it turns out that an increase in the variance of the growth rate of the money stock has the same effect as a decrease in the expected rate of change of the price of money.

## 2. The Economy

The model developed here extends the model of Cox/Ingersoll/Ross (1978) which is widely used in finance. We turn to a finance model because we want to show how households choose the distribution of the output rate for a given investment opportunity set and for given dynamics of the investment opportunity set. The model built here differs from the model developed by Cox/Ingersoll/Ross (1978) because (a) households hold real balances and (b) there is a government sector whose policies change stochastically through time.<sup>11</sup>

We look at an economy in which there is only one commodity. The existing stock of capital,  $k$ , can be invested in  $n$  production processes. The only input required to produce the commodity is the commodity itself.  $K_i$ ,  $i=1, \dots, n$ , is the quantity of the commodity invested in the  $i^{\text{th}}$  production process. The instantaneous output of the  $i^{\text{th}}$  production process is a random variable given by:

$$dK_i = \mu_{K_i} K_i dt + \sigma_{K_i} K_i dz_{K_i} \quad \forall i \quad (12)$$

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<sup>11</sup>For models which incorporate real balances in the Cox/Ingersoll/Ross (1978) model, see Jones (1980), Eaton (1981), Gertler/Grinols (1982), and Stulz (1984). See also Lucas (1984) for a discrete-time alternative model.

where  $dz_{K_i}$  is the increment per unit of time of a standard Wiener process.<sup>12</sup> In this economy, the expected output rate and the variance of the output rate from a production process can change as the state of the world changes. The state of the world for households is described by a  $1 \times s$  vector  $\underline{s}$  of state variables; each state variable is assumed to follow an Ito process so that the endogenous variables also follow an Ito process. Equation (12) implies that production processes exhibit constant returns to scale.

We want to consider a model in which the government does not rebate the proceeds from money creation to the private sector, so that changes in the money stock affect the real wealth of households. If the proceeds from money creation are wholly rebated to households, the results obtained in the remainder of this paper will not hold, because at least for the logarithmic utility function, changes in monetary policy will not affect the households' real wealth. To the extent that changes in the money stock lead partially to transfers, one can view the analysis of this paper as applying to the effects of changes in the money stock net of transfers. In the whole analysis, we take the actions of the government as being exogenously given. The government owns a stock of the commodity, which changes over time because of production and consumption of the commodity by the government and because of open-market operations. It is assumed that the money stock can change only because of open-market operations involving the commodity and money. There are no government bonds.

The money supply  $M$  is assumed to be a function of the state of the world,  $M=M(\underline{s},t)$ . The instantaneous change in the money supply can be obtained by differentiating  $M(\underline{s},t)$  using Ito's Lemma. To simplify the analysis, we study only one type of monetary policy. We assume that the government tries to make the price of money  $\pi$  grow at a rate  $\mu_\pi$ . This is captured by the assumption that  $\pi$  follows an Ito process. The variance of the growth rate of  $\pi$  is  $\sigma_\pi^2$ . The type of monetary policy considered here simplifies the analysis as the instantaneous conditional distribution of the rate of change of  $\pi$  captures all the relevant information about the effects of monetary policy on the price of money over the next instant of time. A more complete analysis would derive the dynamics for  $M(\underline{s},t)$  which,

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<sup>12</sup>Cox/Ingersoll/Ross (1978) provide references on Ito processes and optimal control. For the readers who are not familiar with these techniques, Fischer (1975) provides a useful introduction.

given the dynamics for the quantity of money demanded by households, yield the assumed dynamics for the price of money. However, such a derivation is not discussed here, as it adds nothing to the main results of this paper. To capture the effect of changes in the monetary policy regime,  $\mu_\pi$  and  $\sigma_\pi^2$  are assumed to be functions of the vector of state variables.

### 3. The Households' Optimization Problem

We assume that all households are the same, are infinitely-lived and maximize:

$$E_t \int_t^{\infty} e^{-\rho\tau} \frac{1}{\delta} [c(\tau)^\alpha m(\tau)^{1-\alpha}] d\tau$$

where  $c(\tau)$  is the consumption rate of the commodity and  $m(\tau)$  corresponds to the holdings of real balances.

The households' instantaneous utility function implies constant relative risk tolerance and constant expenditure shares. The real wealth of households is  $w$ .  $n_i$  is the fraction of the households' wealth invested in risky asset  $i$ . Households can invest in  $n+1$  different assets whose return is risky in real terms and in one asset whose instantaneous real rate of return is nonstochastic and equal to  $r$ .  $n$  risky assets consist of investments in production processes. We assume that all markets are perfect.<sup>13</sup> Consequently, the instantaneous return of investments in production processes is given by equation (12). One risky asset is a bond with a safe instantaneous nominal rate of return equal to  $R$ .

We take the first risky asset to have a real rate of return equal to the real rate of return of the nominal bond, which is:

$$\frac{dI_1}{I_1} = Rdt + \mu_\pi dt + \sigma_\pi dz_\pi \quad (13)$$

where  $\mu_\pi$  is the expected growth rate of  $\pi$ .

The household's optimal consumption and portfolio policies must be

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<sup>13</sup>More specifically, we assume that there are no transaction costs, no taxes, and that there is perfect competition.

such that the following flow budget equation is satisfied:

$$dw = \sum_{i=1}^{n+1} n_i \left( \frac{dI_i}{I_i} - rdt \right) w + rwdt - Rm dt - c dt \quad (14)$$

where  $dI_i/I_i$  is the instantaneous rate of return of the  $i^{\text{th}}$  asset. In equation (14),  $n_1$  corresponds to the fraction of the households' wealth invested in the nominal bond plus  $m/w$ . This reflects the fact that the nominal bond and the real balances have perfectly correlated real returns, and consequently, cannot be treated as distinct assets when one forms the variance-covariance matrix of asset returns.<sup>14</sup>  $Rm$  is the opportunity cost of holding real balances.

In this model, households solve for the optimal portfolio of investments knowing the true distribution of asset returns and state variables. Solving for this portfolio, we get:

$$\underline{n} = \left( \frac{T}{c_w w} \right) \underline{V}^{-1} (\underline{\mu} - r \cdot \underline{1}) + \underline{V}^{-1} \underline{V}_{as} \left[ \frac{-c_s}{c_w w} + \left( \frac{m(1-T^R)}{c_w w} \right) \right] \quad (15)$$

where  $T$  ( $T^R$ ) is the coefficient of absolute (relative) risk tolerance of the households' indirect utility function of consumption expenditures. Consumption expenditures  $c = q + Rm$  are given by the function  $c(w, \underline{s}, t)$  and  $c_w$  is the partial derivative of the function  $c(w, \underline{s}, t)$  with respect to  $w$ . The vector  $c_s$  is the  $1 \times s$  vector of partial derivatives of  $c(w, \underline{s}, t)$  with respect to the state variables.  $\underline{V}$  is the  $(n+1) \times (n+1)$  variance-covariance matrix of asset returns,  $\underline{V}^{-1}$  is the inverse of the matrix  $\underline{V}$  and  $\underline{V}_{as}$  is the  $(n+1) \times s$  covariance matrix of asset returns with changes in state variables. The first state variable is taken to be the opportunity cost of holding real balances.  $\underline{0}$  is a  $1 \times (s-1)$  vector of zeros.

#### 4. Asset Demands and Monetary Policy Uncertainty

In this model, the portfolio households choose to hold determines the distribution of the output rate. With a non-stochastic investment oppor-

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<sup>14</sup>See Fama/Farber (1979) for a discussion of holdings of real balances when changes in the price level are stochastic.

tunity set, i.e., the state variables given by the vector  $\underline{s}$  do not change stochastically over time, households hold a mean-variance efficient portfolio whose weights are proportional to  $\underline{v}^{-1}(\underline{\mu} - r.1)$ . This means that they choose the technologies in usage in such a way that no mix of technologies and real balances yields a portfolio whose return dominates in mean-variance space the return of the portfolio they choose to hold.

If the state variables change stochastically over time, the households hedge against unanticipated changes in state variables if their coefficient of relative risk aversion exceeds one. This means that households choose to hold a portfolio such that an unanticipated change in a state variable affects their lifetime expected utility to a smaller extent than if they hold a mean-variance efficient portfolio. However, for households to be able to hedge, the returns of risky assets must be correlated with changes in state variables. Furthermore, households do not hedge if they have a logarithmic utility function, as the logarithmic utility function implies myopic portfolio and consumption policies<sup>15</sup>.

When households hedge against unanticipated changes in monetary policy, monetary policy uncertainty can affect the investment policies. The intuition behind this result can be explained in the following way. If households know that the  $i^{\text{th}}$  state variable affects monetary policy in a way which decreases their lifetime expected utility, they try to hedge against unanticipated changes in state variable  $i$  if their degree of relative risk aversion is high enough. If some asset's return is positively correlated with changes in state variable  $i$ , sufficiently risk-averse households are likely to invest less in this asset than they would otherwise, so that unanticipated changes in their wealth partly offset the effect on their lifetime expected utility of changes in the  $i^{\text{th}}$  state variable. As this asset can be an investment in a production process, the fact that households try to protect themselves against the adverse effects of unanticipated changes in monetary policy can affect the households' choice of technologies to produce the commodity.

It is useful to illustrate this discussion with a concrete example. For the purpose of this example, we assume that there is only one state variable, the expected rate of change of the price of money,  $\mu_{\pi}$ . Not unrealistically, the dynamics of  $\mu_{\pi}$  are assumed to be such that an

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<sup>15</sup>See Breeden (1983)

unexpected increase in the price of money implies an increase in  $\mu_\pi$ :

$$d\mu_\pi = \Omega \left( \frac{d\pi}{\pi} - \mu_\pi dt \right). \quad (16)$$

$\Omega$  is assumed to be positive, but smaller than one. To simplify this example even further, it is useful to assume that (A.1.) unanticipated changes in  $\pi$  are uncorrelated with the returns of production processes and with changes in state variables other than  $\mu_\pi$  and  $R$  and (A.2.) unanticipated changes in  $\mu_\pi$  are perfectly negatively correlated with changes in  $R$ .

It is interesting to note that our example embodies two important stylized facts of modern finance. First, in this economy, we can define an exact price index for households, as their instantaneous utility function exhibits constant expenditure shares.<sup>16</sup> Using the commodity as the numeraire, the price index  $p$  evolves according to:

$$\frac{dp}{p} = (1-\alpha) \frac{dR}{R} \quad (17)$$

Consequently, because changes in  $R$  are positively correlated with inflation, the returns in terms of the commodity of a well-diversified portfolio of common stocks deflated using the price index  $p$  are negatively correlated with inflation. Nelson (1976), Fama/Schwert (1977), and Schwert (1981), among others, show that a well-diversified portfolio of common stocks has a real return, i.e., a return deflated by the Consumer Price Index, negatively correlated with the rate of inflation. Fama (1976) and others have provided evidence that nominal interest rates are an increasing function of expected inflation, which is the case when assumption (A.2) holds.

With the assumptions we have made for this example, the demand for nominal assets can be written as:

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<sup>16</sup>See Samuelson/Swamy (1974).

$$n_1 = \left(\frac{T}{c_{wW}}\right) \left(\frac{R + \mu_\pi - r}{\sigma_\pi^2}\right) - \left[\frac{m(1-T^R)}{c_{wW}} - \frac{c_{1nR}}{c_{wW}}\right] \quad (18)$$

where the state variable is taken to be the log of  $R$ . From assumption (A.2), changes in the log of  $R$  are perfectly negatively correlated with changes in the log of  $\mu_\pi$ .

With the type of policy uncertainty postulated here, an unanticipated fall in  $\pi$ , i.e., the price of one unit of money, brings about a decrease in  $\mu_\pi$  and, consequently, an increase in  $R$ . A household is in worse condition from an increase in the nominal rate of interest because it increases the cost of the services of real balances. A household wants to hedge against the adverse effects of an unanticipated increase in  $R$  if its coefficient of relative risk tolerance is smaller than one, i.e.,  $T^R < 1$  and can do so by taking a short position in nominal assets. If  $\pi$  falls unexpectedly, the household makes an unanticipated gain on its short position in nominal assets which, if the household is fully hedged, offsets the loss it makes because of the increase in  $R$  which accompanies a fall in  $\pi$ . Finally, an unanticipated fall in  $\pi$  corresponds to an unanticipated worsening of the investment opportunity set, as it decreases the expected return from holding real balances. Hence, as it is widely believed that  $T^R < 1$ , one expects  $c_{1nR}$  to be negative, and consequently, a household also takes an additional short position in nominal assets to hedge against an unexpected worsening of the investment opportunity set.<sup>17</sup>

This example implies that, because monetary policy uncertainty creates uncertainty about future investment and consumption opportunities, it decreases the demand for nominal assets and, consequently, increases the nominal rate of interest. While the example just discussed points out how policy uncertainty can affect asset demands, one should not forget that the specific results of this example follow from a number of restrictive assumptions.

## 5. Nominal Interest Rates and Real Balances

In this model, real balances held by households as well as the nominal rate of interest  $R$  are endogenous variables. In this section, we show how these two variables are determined. To be able to characterize our

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<sup>17</sup>For a discussion of what hedging means in a model like this, see Breeden (1984).

solution more precisely, we assume that households have a logarithmic utility function (i.e.,  $\delta=0$ ).<sup>18</sup> Furthermore, we assume that (A.1') the real returns on production processes are uncorrelated with changes in  $\pi$ . With a logarithmic utility function, households do not hedge. Consequently, our results do not require restrictions on the nature of the state variables. The demand for nominal assets is now given by:

$$n_1 w = \left( \frac{1}{2} \right) \frac{(R + \mu_\pi - r)w}{\sigma_\pi} . \quad (19)$$

Notice that the demand for nominal assets given here holds irrespectively of the assumptions made about the state variables or about the dynamics of  $\mu_\pi$ , as long as the state variables and  $\mu_\pi$  follow Ito processes. Because the logarithmic utility function simplifies the asset demands, it makes it easier to characterize the equilibrium value of endogenous variables in a general equilibrium rational expectations model.

In equilibrium, it must be true that the real value of nominal assets held by households must be equal to the real balances they hold, as there are no nominal bonds. Consequently, we have:

$$n_1 w = m. \quad (20)$$

Furthermore, as consumption expenditures are equal to  $\rho w$  in this case, it follows that:

$$\rho(1-\alpha)w = Rm. \quad (21)$$

However, the real wealth of households is equal to the sum of the stock of capital,  $k$ , and of monetary wealth,  $m$ , i.e.,  $w=m+k$ . Hence, equation (21) can be rewritten as:

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<sup>18</sup>The expected indirect utility function of wealth is given in this case by:

$$J(w, \underline{s}, t) = (1/\rho) e^{-\rho t} \{ n w + G(\underline{s}, t) \} .$$

See, for instance, Cox/Ingersoll/Ross (1978) for a derivation of the expected indirect utility function of wealth in a general equilibrium setting.

$$R = \rho(1-\alpha) \left\{ \frac{k}{m} + 1 \right\} \quad (21')$$

Equations (19) and (21') can be viewed as a system of two equations in two unknowns, R and m. Rewriting equation (19), we have:

$$\sigma_{\pi}^2 \left( \frac{m}{m+k} \right) - \mu_{\pi} + r = R \quad (19')$$

Consequently, for a given real rate of interest, equations (19') and (21') yield R as a function of m. Figure 2 plots these two functions. Equation (20') shows the relation which must exist between R and m for portfolio equilibrium. Equation (21') shows how R and m must be related to satisfy the first-order condition that the marginal utility of real balances must be equal to R times the marginal utility of real wealth. On Figure 2,  $m^*$  and  $R^*$  are the equilibrium values of m and R.

Figure 2 can be used to study the effect on the nominal rate of interest of (a) an increase in the real rate of interest r, (b) an increase in the expected rate of change of the price of money  $\pi$ , and (c) an increase in the variance of the rate of change of  $\pi$ . As r,  $\mu_{\pi}$  and  $\sigma_{\pi}^2$  all depend on the vector of state variables  $\underline{s}$ , they may change continuously in this model. A change in  $\mu_{\pi}$  or  $\sigma_{\pi}^2$  corresponds to a change in monetary policy. Such a change is shown to affect the real rate of interest in the next section. The real rate of interest can also change because of changes in the technologies. However, the analysis of the remainder of this section and of the next section is carried out under the assumption of constant technologies to highlight the effects of changes in monetary policy.

In equation (19'), the effect of an increase in r is the opposite of the effect of an increase in  $\mu_{\pi}$  for a given level of real balances. Given m, a change in  $\mu_{\pi}$  or r has no effect on the right-hand side of equation (21'). Consequently, in Figure 2, an increase in r or a decrease in  $\mu_{\pi}$  creates a vertical shift in the curve given by equation (19') and does not affect the curve given by equation (21'). It follows that an increase in r or a decrease in  $\mu_{\pi}$  decreases the real balances held by households and increases the nominal rate of interest.

Notice that, for a given nominal rate of interest R, an increase in r or a decrease in  $\mu_{\pi}$  decreases the risk premium paid on nominal assets, i.e., decreases  $R + \mu_{\pi} - r$ . An increase in R must follow an increase in r or a fall in  $\mu_{\pi}$  to reestablish the risk premium at its earlier level. By the same token, an increase in  $\sigma_{\pi}^2$  decreases the risk premium on nominal

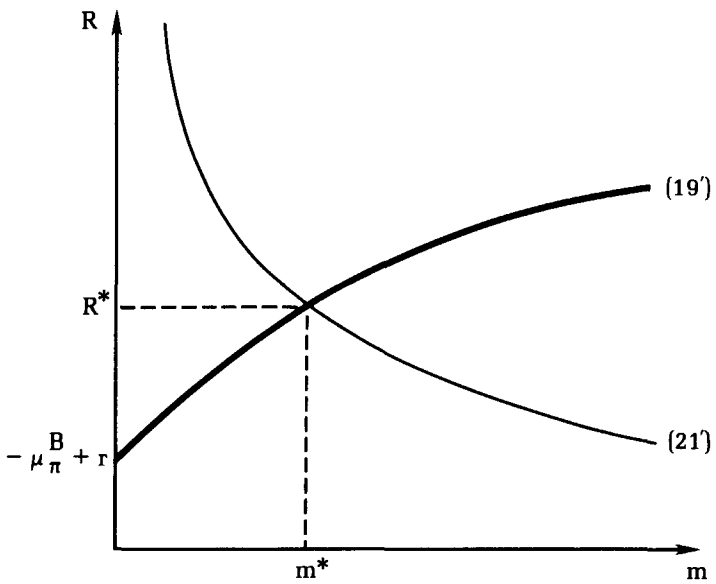


Figure 2

Equation (19') characterizes portfolio equilibrium for households  
 Equation (21') relates holdings of real balances to consumption expenditures  
 $R^*$  corresponds to the equilibrium interest rate  
 $m^*$  corresponds to the equilibrium holdings of real balances for a given amount of real wealth

assets per unit of variance, and consequently, the nominal rate of interest must rise to bring the risk premium back to its earlier level. This analysis of the effect of an increase in  $\sigma_{\pi}^2$  does not hold if the risk premium on nominal assets is negative. If nominal assets have real returns negatively correlated with the output rate, holding nominal asset may decrease the variance of the real rate of return of invested wealth. In this case, the risk premium on nominal assets is negative.

## 6. The Real Rate of Interest and Macroeconomic Fluctuations

In this section we show how changes in monetary policy affect the real rate of interest  $r$  and the distribution of the output rate. We focus the discussion on the simple case in which unanticipated changes in  $\pi$  are uncorrelated with the returns of production processes.

First, we derive the effect of a change in monetary policy on the real rate of interest, i.e., the instantaneous real rate of return on an asset which promises a safe, instantaneous payoff. Let the subscript  $e$  denote that the first row and column have been taken out of a matrix. With assumption (A.1'), the households' holdings of investments in production processes can be written:

$$\underline{n}_e = \underline{V}_e^{-1} (\underline{u}_e - r \cdot \underline{1}) \quad (22)$$

Equilibrium requires that the whole stock of capital be invested in production processes:

$$\underline{1}' \underline{n}_e w = k \quad (23)$$

Using equations (22) and (23), we get:

$$k = \underline{1}' \underline{V}_e^{-1} (\underline{u}_e - r \cdot \underline{1}) w \quad (24)$$

Using the fact that  $w = m + k$ , we can rewrite equation (24) to obtain the real rate of interest as a function of  $m$  and  $k$ :

$$r = \frac{a(m+k) - k}{b(m+k)} \quad (25)$$

where  $a = \underline{1}' \underline{V}_e^{-1} \underline{u}_e$  and  $b = \underline{1}' \underline{V}_e^{-1} \underline{1}$ .  $a$  and  $b$  are exogenously given. They

change stochastically over time only if the distribution of the returns of some production process changes over time.

Differentiating equation (25), we get:

$$\frac{dr}{dm} = \frac{k}{b(m+k)^2} > 0 \tag{26}$$

Equation (26) means that the real rate of interest is higher in states of the world in which real balances are higher. As an increase in the nominal rate of interest decreases  $m$ , it follows that an increase in the variance of the rate of growth of  $\pi$  or a decrease in the expected rate of growth of  $\pi$  decreases the real rate of interest. Consequently, in this model a fall in the real rate of interest is associated with a fall in the lifetime expected utility of households. While this result seems paradoxical in the light of much of the macroeconomic literature, it has a simple explanation. In this model, production decisions are endogenous. With constant relative risk aversion, households invest a constant fraction of their wealth in production processes for a given real rate of interest. An increase in the nominal rate of interest decreases the households' holdings of real balances and, hence, their real wealth. For a constant real interest rate, households want to invest less in production processes and more in the nominal bond following an increase in  $R$ . To understand why households want to invest more in the nominal bond, notice that, for a given real rate of interest, the households' demand for nominal assets falls by less than their holdings of real balances. However, in equilibrium, there can be no investment in the nominal bond. Consequently, the real rate of interest must fall following an increase in the nominal rate of interest to make the nominal bond less attractive, so that no household wants to invest in that asset.

We now consider the effect of a fall in the real rate of interest on the expected output rate  $\mu_k = E(dk/k)$ . Notice that the expected output rate is given by  $\underline{n}'_e \underline{\mu}_e (w/k)$ , which corresponds to the expected output divided by the stock of commodity  $k$ . Using equation (23), which defines the vector  $\underline{n}_e$ , the expected output rate can be obtained as a function of exogenous variables and of the real rate of interest  $r$ :

$$\mu_k = \frac{\underline{\mu}'_e \underline{V}_e^{-1} (\underline{\mu}_e - r \cdot \underline{1})}{\underline{1}'_e \underline{V}_e^{-1} (\underline{\mu}_e - r \cdot \underline{1})} \tag{27}$$

Differentiating the expected output rate with respect to  $r$ , we obtain:

$$\frac{d\mu_k}{dr} = \frac{(\frac{1}{\mu_e} - 1)(\frac{1}{\mu_e} - 1) - (\frac{1}{\mu_e} - 1)(\frac{1}{\mu_e} - 1)}{(\frac{1}{\mu_e} - 1)(\mu_e - r - 1)^2} \quad (28)$$

Merton (1972) proves that the numerator of the expression on the right-hand side of equation (28) is positive. Consequently, the expected output rate is positively related to the real rate of interest. This relation between the expected output rate and the real rate of interest is well-understood in models without money (see Breeden (1984)). In these models, an adverse change in technologies makes the riskless asset more attractive and, consequently, the real rate of return on that asset must fall to maintain equilibrium. In the present model, a change in monetary policy, which causes the real wealth of households to fall, implies that they are no longer willing to bear the same amount of risk as they did before the change in monetary policy. For an unchanged real rate of interest and unchanged portfolio shares for investments in technologies, part of the commodity stock would no longer be invested in production. In equilibrium, the whole stock of capital must be invested in production. Consequently, households must shift to a less risky production plan, so that none of the capital stock is left idle. As the variance of the output rate falls, the expected output rate must also fall as, otherwise, the households' previous production plan was not efficient in the sense that households would have been able to obtain higher expected output for less risk. Therefore, a decrease in the expected growth rate of the price of money is accompanied by a decrease in the expected output rate and in the variance of the output rate. As the nominal rate of interest is an increasing function of the variance of the rate of change of the price of money, an increase in that variance has the same effects as a decrease in the expected rate of change of the price of money.

In this model, the dynamics of the expected output rate are a function of the dynamics of the technologies and of monetary policy, as can be noted by inspection of equation (26). By a proper specification of the dynamics of the technologies and of monetary policy, it is possible to construct examples such that the distribution of the output rate is the same as the distribution of the time-series of the output rate. For instance, suppose that there is only one state variable which is the expected rate of change of the price of money,  $\mu_\pi$ . If  $\mu_\pi$  follows a martingale, then the expected

output rate follows a martingale also. Furthermore, if the variance of changes in  $\mu_\pi$  is high enough, changes in the expected output rate account for a larger fraction of changes in the output rate than the deviations of the output rate from its mean. One can also construct examples in which all the variation in the expected output rate can be attributed to changes in technologies or examples in which both the changes in technologies and changes in monetary policy contribute to explaining the changes in the expected output rate.

#### IV. CONCLUDING REMARKS

The empirical analysis presented in this paper shows that, in general, one should decompose macroeconomic time-series into a stochastic trend, a cyclical, and a seasonal component rather than into a deterministic trend, a cyclical, and a seasonal component. If a stochastic trend is present in a time-series, the autoregressive representation of that time-series exhibits a first-order unit root. We examine monthly and quarterly time-series for a number of countries and are unable to reject the hypothesis that the autoregressive representation of these time-series exhibits a first-order unit root. An important implication of these results for empirical work is that first differencing of the data is required to get a stationary time-series.

However, if the decomposition involving a stochastic trend is used, much of the business cycle turns out to be a statistical artifact. In particular, changes in the trend component seem to account for more variation in macroeconomic time-series than the cyclical component. Consequently, with the stochastic trend decomposition, it is crucial to explain changes in the trend of macroeconomic time-series if one wants to explain macroeconomic fluctuations. We present a theoretical model in which the dynamics of output have the same properties as the macroeconomic time-series in our empirical study. It is shown that changes in the trend of output can be caused by changes in real variables usually considered by growth theory as well as by changes in macroeconomic policies. In the model, macroeconomic policies affect the distribution of output through their effect on the portfolio of assets held by households. An increase in the variance of the growth rate of the money stock, for instance, increases the nominal rate of interest and decreases the households' real wealth. When the households' real wealth falls, households choose to bear a smaller

amount of production risk. Consequently, an increase in the variance of the growth rate of the money stock causes a decrease in the variance of output and in the expected value of output.

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