

THE FORWARD EXCHANGE RATE AND MACROECONOMICS

René M. STULZ*

Graduate School of Management, University of Rochester, Rochester N.Y. 14627, U.S.A.

Received September 1980, revised version received August 1981

This paper discusses the conditions under which a risk premium is incorporated in the forward exchange rate. A new condition for the existence of a risk premium is proposed. We show that earlier models of the risk premium, which emphasize either the role of net foreign investment or of the relative supplies of 'outside' assets, are not suited for assessing the effects of changes in macroeconomic policy.

1. Introduction

Much work has been done recently on the question of whether or not the forward exchange rate (domestic price of the foreign currency on the forward market) is an unbiased predictor of the future spot rate. In equilibrium, the forward exchange rate is a biased predictor of the future spot rate if it contains a risk premium.¹ The conventional wisdom, which argues that the risk premium on the forward exchange rate has something to do with the net foreign investment of the domestic country, has been strongly challenged.² A new line of thought implies that this risk premium depends on the outside supply of nominal assets.³ In a study which is representative of this new approach, Frankel (1979) shows that:

*Financial help from the Swiss National Research Fund and from the Center for Research in Government Policy and Business is gratefully acknowledged. I thank Robert Cumby, Donald Lessard, Charles Plosser, Patricia Reagan, and Lee Wakeman for useful comments.

¹The risk premium is defined as the expected gain required in equilibrium for domestic investors to be willing to hold the foreign currency long. If the expected future spot exchange rate is maintained constant, an increase in the risk premium means that the expected return on holding foreign bonds increases and that the forward exchange rate falls. A fall in the forward exchange rate means that, *ceteris paribus*, the expected gain from buying the foreign currency forward increases. To avoid issues which are not relevant to the purpose of this paper Siegel's Paradox [see Siegel (1972)] is neglected. If the risk premium is positive, the forward exchange rate is lower than the expected future spot exchange rate.

²Frankel (1979) gives references to that conventional wisdom. Solnik (1973) offers a finance-theoretic model in which the risk premium is an increasing function of net foreign investment. See Stulz (1980) for an approach which yields such a result.

³See Frankel (1979). Those conclusions can also be found in other papers, in particular Kouri (1977), Fama and Farber (1979) and Grauer, Litzenberger and Stehle (1976). We focus here on Frankel's paper because the paper is extremely lucid and uses an approach which is familiar to economists.

(1) if there are 'outside' nominal assets, the forward rate contains a risk premium, and hence is a biased predictor of the future spot rate; and

(2) if the exchange rate is correlated with the value of real assets, again there is a risk premium.

Those results have important macroeconomic implications, since a risk premium on the forward exchange rate that depends on the supply of 'outside' assets implies that a policy which changes the supply of 'outside' assets can change the real interest rate in one country relative to the real interest rate in another country.⁴

The present paper re-examines both the conventional and the new wisdom on the risk premium contained in the forward exchange rate. A general result on the risk premium is presented and the role of exchange rate dynamics in the determination of the risk premium is discussed. The conclusions of the paper are as follows.

1. If the real exchange rate changes through time, the forward rate may contain a risk premium in the absence of 'outside' assets or of a correlation between the exchange rate and the value of real assets. A particular case of stochastic changes in the real exchange rate occurs when all movements of the exchange rate are changes in the real exchange rate. For that particular case, it can be shown that the risk premium contained in the forward rate is always an increasing function of the stock of net foreign assets of the domestic economy.

2. Exchange rate dynamics which imply that today's exchange rate is correlated with the expected rate of change of the exchange rate or with the expected returns of some risky assets, in general imply the existence of a risk premium on the forward exchange rate, even if Frankel's conditions for a risk premium are not met and if the real exchange rate does not change through time.

3. A general result about the risk premium contained in the forward exchange rate is that, in a world in which there are no obstacles to international capital flows,⁵ the risk premium is an increasing function of the correlation of changes in the domestic exchange rate with changes in world real consumption. If the exchange rate is correlated with the value of real assets, it is also correlated with world real consumption, whereas the converse is not true.

The plan of the paper is as follows. In section 2 a definition of risk is presented and used to derive Frankel's results in a simple way. In section 3 it is shown that this definition of risk is inappropriate when the real exchange rate changes stochastically through time. The implications of stochastic

⁴See Dornbusch (1980).

⁵Finance-theoretic models of barriers to international investment are given, for instance, by Black (1974), Adler and Dumas (1976), Kouri (1976) and Stulz (1981b).

changes in the real exchange rate for the forward exchange rate are also discussed. In section 4 risks induced by macroeconomic policies are discussed and shown to be generally inconsistent with the simple definition of risk used in the second section. In section 5 a general result on the risk premium contained in the forward exchange rate is presented. Finally, in section 6 some concluding remarks are offered.

2. Exchange rates and risky assets

We make the simplifying assumption that there is one commodity. If there is only one commodity, i.e. if the real exchange rate is constant, the real return of an asset is simply the return of that asset when the commodity is used as the numeraire. Assume further that all investors act as if they are maximizing a utility function which is an increasing function of expected end-of-period real wealth and a decreasing function of the variance of end-of-period real wealth. For simplicity, we make the additional assumption that all investors are the same, in the sense that they have the same utility function and the same initial real wealth. This assumption allows us to restrict our attention to a representative investor.

In the economy considered, there are N investors, n assets which are risky in real terms, and one safe real asset, i.e. a bond which promises the payment of a certain number of units of the commodity next period. Define w_i as the fraction of his real wealth W the representative investor invests in the i th asset. The vector w , whose representative element is w_i , characterizes the portfolio held by the representative investor. By definition, if e' is a row vector of ones, then $e'w$ is equal to one. Let w^a and w^b be two feasible portfolios. When can we say that portfolio w^a is more risky than portfolio w^b ? Since, by assumption, investors are only concerned with the expected return of a portfolio and its variance, a natural definition of risk is that portfolio w^a is more risky than portfolio w^b if the portfolios w^a and w^b have the same expected real return but portfolio w^a has a larger variance than portfolio w^b .⁶

Given the distribution of asset returns, the investor will choose a portfolio which, among all portfolios which have the same expected real return, is the least risky. Let w^* be the optimal portfolio for the investor. Define w^s as the vector whose representative element w_i^s , $i=1, \dots, n$, is the supply of the i th asset expressed as a fraction of world real wealth. In other words, w_i^s is the proportion of world real wealth held in asset i . By definition, it must be the case that $e'w^s=1$. Since all investors are the same, market equilibrium requires $w^*=w^s$.

⁶With the assumed utility function, this definition of risk corresponds to the one of Rothschild and Stiglitz (1970).

How will securities be priced when market equilibrium has obtained? Clearly, securities must be priced so that no portfolio which has a smaller variance than w^s can have a higher expected rate of return than w^s . It follows that a marginal change in portfolio w^s which consists in increasing slightly the holdings of asset i and decreasing slightly the holdings of asset j should decrease the expected return of the portfolio whenever that marginal change decreases the variance of the portfolio. Let σ_s^2 be the variance of portfolio w^s and σ_{ij} be the covariance of asset i and asset j . The change in the variance of portfolio w^s is given by:

$$\begin{aligned} \left(\frac{\partial \sigma_s^2}{\partial w_i} - \frac{\partial \sigma_s^2}{\partial w_j} \right) &= 2 \sum_{k=1}^n w_k^s \sigma_{ik} - 2 \sum_{k=1}^n w_k^s \sigma_{jk} \\ &= 2(\sigma_{is} - \sigma_{js}), \end{aligned} \quad (1)$$

where σ_{is} is the covariance of asset i and portfolio w^s . It follows that if $\sigma_{is} > \sigma_{js}$, the expected rate of return on asset i must be larger than the expected rate of return on asset j .

Thus far we have been concerned only with real returns. However, we made no assumption that excludes an asset with a safe nominal return, i.e. an asset which pays a fixed number of units of money at the end of the period. Let asset l be an asset that has a safe nominal return in terms of the domestic currency, and let asset M be an asset that has a safe nominal return in terms of the foreign currency. To simplify the discussion, we assume that assets l and M are the only assets which have safe nominal returns.

Clearly, these additional assumptions do not affect our earlier discussion. When there is only one commodity, i.e. when purchasing power parity always holds exactly, and when there are no barriers to international investment, differences in the expected real rates of return of the safe nominal bonds can arise only from differences in price level dynamics across countries. If the expected real rate of return on both nominal bonds is the same, no risk premium will be incorporated in the forward exchange rate. A necessary condition for both nominal bonds to have the same expected real rate of return is that the real rate of return of each nominal bond has the same covariance with the real rate of return on portfolio w^s , i.e. we need that:

$$w_l^s \sigma_l^2 + \sum_{j \neq l} w_j^s \sigma_{lj} = w_M^s \sigma_M^2 + \sum_{i \neq M} w_i^s \sigma_{iM}. \quad (2)$$

Note that the variance and covariance terms in (2) correspond to covariances and variances of real returns. (2) will hold in the trivial case in which both price levels are perfectly correlated. If all assets $k \neq l$ and $k \neq M$, such that $w_k^s \neq 0$, are common stocks, a sufficient condition for the absence of a risk

premium in the forward rate is that: (a) inflation rates are not correlated with the return on the world market portfolio of common stocks, and (b) there are no safe 'outside' nominal assets, in the sense that $w_f^s = 0$ and $w_M^s = 0$.

In this section, the covariance of an asset with portfolio w^s is a natural measure of the risk of that asset. If an asset which is perfectly correlated with the exchange rate is risky, in the sense that it has a non-zero covariance with the portfolio w^s , the asset will have a positive risk premium if it is positively correlated with w^s and a negative risk premium if it is negatively correlated. A more formal analysis can be used to show that the assumption that all investors are alike does not affect these results. The assumption that investors care only about the expected terminal wealth and its variance can also be relaxed considerably.⁷

Only two assumptions, which have great economic significance, seriously affect the generality of the measure of risk used in this section. The first assumption, which has been made explicitly, is that the economy can be represented as an economy with only one commodity. The second assumption, which has been made implicitly, is that there exists no asset whose return is correlated with changes in the vector of expected returns.⁸ Without those two assumptions the riskiness of a security cannot be defined as its contribution to the variance of the real return of a portfolio, in the sense that if two securities are compared the one with the highest covariance with the world market portfolio is not necessarily the security with the highest expected real rate of return. In section 3 we remove the assumption which states that there is only one commodity, whereas in section 4 we motivate the second crucial assumption and discuss how the risk premium contained in the forward exchange rate is affected when it is relaxed.

3. Stochastic changes in the real exchange rate

In this section we are interested in the effect of the existence of many commodities on the risk premium incorporated in the forward exchange rate. Because our goal in this section is limited, we can use an extreme situation: each country produces only one good. To simplify the problem, let us assume that in each country the produced commodity is the numeraire and that the nominal price of one unit of each commodity is constant in the country in which it is produced. It follows that in this world all exchange rate changes are real exchange rate changes.

⁷Kouri (1977) provides a model which does not require such a strict assumption. The model presented here is a version of the capital asset pricing model popular in the finance literature; see Sharpe (1964) and Lintner (1965).

⁸Hellwig (1977) and Constantinides (1980) discuss how acceptable that assumption is in closed economies.

Several assumptions will facilitate our analysis. First, let the proportion of consumption expenditures that an investor i spends on the commodity produced in the domestic country be a constant α_i .⁹ Secondly, all domestic investors are the same, and all foreign investors are the same. Thirdly, the assets available are common stocks and nominal bonds. In such a world, an investor desiring to guarantee himself a certain consumption flow in each commodity would simply hold his wealth in two assets. Let $\alpha_i = \alpha$, for all domestic investors. A safe real asset for a domestic investor consists of α units of the domestic nominal bond and $1 - \alpha$ units of the foreign nominal bond. In this framework a risk-averse investor may want to hold a positive net amount of foreign bonds because those bonds allow him to reduce the variability of his real consumption flow, i.e. they allow him to hedge against unanticipated changes in the terms of trade.¹⁰

Suppose now that domestic investors consume only domestic goods, i.e. $\alpha = 1$.¹¹ In that case, domestic investors will not hold foreign nominal bonds for the purpose of holding a safe real asset. For domestic investors, the domestic nominal asset is a safe real asset. In such a world, foreign nominal bonds are just like any other asset for domestic investors. A foreign nominal bond is risky for a domestic investor if it contributes to the variance of the real return of his optimally invested wealth. In section 2, because of the assumption that all investors are the same, we could state that all investors invest their wealth in the same way, i.e. they hold the portfolio w^* . The same statement does not hold here. Even if foreign and domestic nominal bonds do not belong in the world portfolio of risky assets, i.e. w^* , domestic investors might want to hold a negative amount of domestic nominal bonds if, for instance, domestic nominal bonds enter in the safe real asset of foreign investors.

An investor bears exchange rate risk if he holds assets which have a non-zero covariance with the exchange rate. Before discussing the conditions under which a domestic investor finds it optimal to bear exchange rate risk, we make the further assumption that no common stock has a non-zero covariance with the exchange rate. This assumption implies that, if the domestic investor holds a foreign stock which is not hedged against unanticipated changes in the exchange rate, he bears exchange rate risk, because the variance of the rate of return in domestic currency of a foreign

⁹If this assumption is not made, the risk premium incorporated in the forward rate depends also on the elasticities of expenditure for imported consumption goods, as shown in Stulz (1980).

¹⁰Heckerman (1973) is to be credited with being the first to have focused attention on the fact that assets perfectly correlated with the terms of trade expand the investment opportunity set of investors. Fischer (1975) has a model of portfolio choice with many commodities for a closed economy. See also Wihlborg (1978).

¹¹This case corresponds to the one studied by Solnik (1973). As shown in Stulz (1980), this assumption is not required to obtain a risk premium which depends on net foreign investment. The assumption that $\alpha = 1$ facilitates the exposition considerably.

stock is (in the limit of continuous-time) the sum of the variance of the rate of return in foreign currency of the stock plus the variance of the percentage change in the domestic-currency price of the foreign currency. If domestic investors are not rewarded for bearing foreign exchange risk, their optimally invested wealth will be fully hedged against unanticipated changes in the exchange rate. Because we assumed that foreign stocks are not correlated with the exchange rate, holdings of foreign stocks by domestic investors are hedged against unanticipated changes in the exchange rate if a domestic investor borrows in foreign currency to buy foreign stocks.

An example helps to motivate the optimality of hedging foreign stocks completely against real exchange rate risk. Suppose that the forward exchange rate is an unbiased predictor of the future spot exchange rate, i.e. the forward exchange rate does not contain a risk premium. We will show below under which conditions this is compatible with asset market equilibrium. Let w^s be the portfolio whose weights correspond to the fractions of world wealth supplied in the various assets. w_l^s and w_M^s correspond, respectively, to the supply of the domestic nominal bond and to the supply of the foreign nominal bond. Clearly, it is possible that $w_l^s=0$ and $w_M^s=0$. Let w^h be the portfolio w^s hedged against foreign exchange risk for a domestic investor. If all assets j , $j \geq M$, are foreign assets, and if $w_l^s = w_M^s = 0$, then $w_j^s = w_j^h$, $\forall j \neq l, M$, and $w_M^s = -\sum_{j>M} w_j^s$. In other words, the domestic investor borrows abroad, in the form of the safe nominal bond, a fraction of his wealth which is equal to the fraction of his wealth he invests in foreign stocks. [Clearly, the domestic investor could attain the same goal of a portfolio fully hedged against foreign exchange risk by using the forward exchange rate market, but this is irrelevant for the present discussion.] Portfolio w^h has no net investment in foreign assets and is perfectly hedged against unanticipated changes in the exchange rate, because a 1 percent increase in the exchange rate increases by 1 percent the value of the investor's debt in foreign currency. Since there is no risk premium, the expected rate of return of portfolio h is the same as the expected rate of return of portfolio s , but the variance of the real rate of return of portfolio h is lower than the variance of the real rate of return of portfolio s , since portfolio h is not correlated with the terms of trade. If the forward exchange rate incorporates a risk premium, the investor's optimally invested wealth will exhibit a correlation with the terms of trade, since the investor is rewarded for bearing that risk. If foreigners consume only foreign goods, they want to hedge their holdings of domestic stocks against exchange rate risk, which means that, in the absence of a forward market, they borrow from domestic investors to buy domestic securities. Because we assumed that all domestic investors are the same and that all foreign investors are the same, there is no borrowing or lending among domestic investors or among foreign investors. Suppose that domestic wealth exceeds the value of domestic common stocks. Therefore, in

equilibrium the domestic country has net positive foreign investment. In the case we are considering, equilibrium can obtain only if domestic investors have net positive holdings in foreign currency assets, i.e. are not completely hedged against unanticipated changes in the terms of trade, and/or foreign investors have net borrowings in domestic currency. Because positive holdings in foreign currency are risky for domestic investors, equilibrium can obtain only if domestic investors are rewarded by a positive risk premium for holding the foreign currency long. If one country has positive net foreign investment, it is not possible for both countries to avoid bearing exchange rate risks. The larger net domestic foreign investment, the lower, *ceteris paribus*, is the forward exchange rate.

Thus far, in this section we have assumed that home-currency prices of common stocks are not correlated with the terms of trade. This assumption can easily be removed. If exchange rate risk can be avoided at zero costs, it is optimal for investors to do so. This means that if investors are not rewarded for bearing exchange rate risk, they want to hold a portfolio which is not correlated with the exchange rate when they only consume the commodity produced in their home country. If all investors are the same, and if there is no net foreign investment, exchange rate risk is priced as in section 2, in the sense that the higher the covariance of unanticipated changes in the exchange rate with the real return of portfolio w^s the larger the exchange rate risk and the smaller the forward exchange rate. The real rate of return on portfolio w^s now depends on the change in the convex combination of the price of two different goods.

Alternatively, suppose we relax the assumption about consumption preferences, in the sense that we allow investors to consume foreign goods without imposing the assumption that all investors are the same. We keep the assumption that investors spend among consumption goods in constant proportions. Clearly, the more domestic investors consume foreign goods, *ceteris paribus*, the more they are willing to hold assets denominated in the foreign currency, which contributes to reducing the risk premium for bearing exchange rate risk.¹² It is more cumbersome to relax the assumption that changes in the exchange rate are perfectly correlated with changes in the real exchange rate, but in that case the risk premium on a foreign nominal bond with respect to the safe *real* asset for domestic investors can be viewed as the weighted sum of two risk premia, which correspond respectively to the risk premium of this section and the risk premium of section 2.

In this section we have given a lot of attention to a case that is the opposite of the one considered by those who stress the importance of the

¹²As shown in Stulz (1980), this is correct provided that the expected *real* returns are maintained constant as consumption of foreign goods is increased.

supply of outside nominal assets. If the exchange rate is correlated with the real exchange rate, a risk premium can be incorporated in the forward exchange rate even if there is no outside nominal asset. The forward exchange rate, *ceteris paribus*, is a decreasing function of the net foreign investment of the domestic country.

4. Intertemporal risks

In this section we return to a world in which there is only one commodity. Let \mathbf{R}_{t+1} be the vector of expected real returns on risky assets for the period from date t to date $t+1$ conditional upon information available at date t . At date t , \mathbf{R}_{t+2} is a random variable, which we write $\mathbf{R}_{t+2}(t)$. Finally, let $\tilde{\mathbf{R}}_{t+1}$ be the vector of real returns such that $E_t(\tilde{\mathbf{R}}_{t+1}) = \mathbf{R}_{t+1}$, where $E_t(\cdot)$ denotes that the expectation is taken conditional on the information available at date t . Models which stress the importance of the outside supply of nominal assets all assume, implicitly or explicitly, that $\text{cov}[\tilde{R}_{t+1,i}, \tilde{R}_{t+2,j}(t)] = 0$, for all i and j , where $\tilde{R}_{t+1,i}$ indicates the i th element of $\tilde{\mathbf{R}}_{t+1}$. In other words, the covariance between this period's return on a risky asset and next period's expected return on *any* risky asset must be equal to zero. In this section we show what happens when this crucial assumption is removed.¹³

In sections 2 and 3 the choice problem of an investor could always be viewed as maximizing an expected indirect utility function which depends only on real wealth. For such a procedure to be used, it is necessary (a) that each investor has a utility function which implies the existence of a well-defined price index, and that (b) whenever his end-of-period real wealth has the same distribution, an investor has the same expected utility — except at most for factors which depend on time. We are not interested here in what happens when condition (a) does not hold, but we want to illustrate why $\text{cov}[\tilde{R}_{t+1,i}, \tilde{R}_{t+2,j}(t)] \neq 0$, for some i and j , implies that condition (b) does not hold.

Suppose that the economy considered has n risky assets whose expected returns can change stochastically through time. Let us look now at an investor who does not want to bear any risks. Clearly, if there is a safe real asset and the investor's life ends at the end of next period, i.e. at the end of period $t+1$, the investor will only buy the one-period safe real asset. Suppose, however, that the investor will live until the end of period $t+2$. If there exists a two-period safe real bond, the investor can invest his wealth in the one-period safe bond and in the two-period safe bond. By using a one-period safe bond and a two-period safe bond the investor can make his consumption non-stochastic until the end of his life. However, the fact that

¹³Merton (1973) offers a model in which that assumption does not hold for a closed economy. There is now an increasing literature which discusses what we call here 'intertemporal risks'.

the investor chooses his consumption to be non-stochastic until the end of his life does not imply that he chooses his real wealth at date $t+1$ to be non-stochastic. Indeed, as of date t , wealth at date $t+1$ is stochastic because the value of the two-period bond at the end of the first period is stochastic, since it corresponds to the discounted value of the end of period two value of the bond at an interest rate which, as of today, is stochastic. If only the distribution of end-of-period real wealth matters, an investor who has no tolerance for risk would choose a portfolio which implies a safe one-period return. In our example the fact that the first period return in a two-period bond is correlated with the second period safe rate of return means that an investor, who has no tolerance for risk, would choose a risky portfolio this period.

In the example just given, it will not be true, in general, that an investor's portfolio will be a portfolio which would be held if the investor acted as if he was maximizing an expected indirect utility function of real wealth. If, however, the two-period bond is not available and no common stock has a return this period which is correlated with next period's vector of expected rates of return, an investor can do nothing to hedge against unanticipated changes in next period's vector of expected returns. The investor will still behave 'as if' he was maximizing an end-of-period expected indirect utility function of real wealth. It follows that a discussion of risks related to changes in next period's vector of expected returns is relevant only if there exist assets whose returns are correlated with those expected returns. It is also important to stress that investors might want to hold assets to hedge against unanticipated changes in variables that are not expected returns but that can affect future consumption. In what follows we call all those variables which affect the consumption plans of investors independently of real wealth *state variables*. In an Arrow-Debreu framework the state variables are simply the variables which characterize the state of the world.

In a world in which investors want to hedge against unanticipated changes in the state of the world, the risk premium contained in the forward rate depends on whether or not this period's change in the exchange rate is correlated with the value of the state variables next period. Suppose that between date t and date $t+1$ a change in macroeconomic policy occurs. We assume, throughout this section, that macroeconomic policies have real effects and, therefore, affect the consumption plans of investors. It follows that macroeconomic policies are state variables. Since investors are risk averse, it is, in general, optimal for them to try to hedge against unanticipated changes in state variables. In general, macroeconomic policies affect exchange rates. If a macroeconomic policy put into effect between date t and date $t+1$ affects the exchange rate at date $t+1$, it follows that a position in a foreign currency will be correlated with the state of the world at date $t+1$. In this case, the demand for foreign nominal assets of domestic

investors is not the same as in the case in which those foreign nominal assets do not allow investors to hedge against unanticipated changes in state variables. Changes in the demand for foreign nominal assets affect the risk premium. It follows that the risk premium incorporated in the forward exchange rate depends very much on the exchange rate dynamics. If the exchange rate dynamics assumed in constructing a model imply a constant expected rate of change for the exchange rate, that model cannot be used to discuss the effect on the risk premium of changes in macroeconomic policies which imply a change in the expected rate of depreciation of a currency. It should be clear, for instance, that all models of exchange rate dynamics which imply that unanticipated changes in macroeconomic policy create large changes in today's exchange rate are not compatible with simple models of the forward exchange rate which assume that investors maximize an expected indirect utility function which depends only on end-of-period real wealth and time.

5. Why is there a risk premium?

In section 2 we presented a simple definition of risk for a security, i.e. a security's risk is proportional to its contribution to the variance of the real return of the market portfolio, and saw that that definition of risk implies that the forward exchange rate is a biased predictor of the future spot exchange rate when there exist 'outside' bonds or when the exchange rate is correlated with the value of real assets. In sections 3 and 4 we showed that this definition of risk is too restrictive to allow us to study some important phenomena. In particular, we stressed that unanticipated changes in the real exchange rate and unanticipated changes in macroeconomic policies could be risky for the investor, in the sense that an investor would be willing to hold a portfolio with a lower expected rate of return if that portfolio is sufficiently hedged against unanticipated real exchange rate or macroeconomic policy changes. It would be useful to have a definition of risk which is sufficiently general and lacks the defaults of the definition given in section 2. It turns out that such a definition exists. The remainder of this section is devoted to a discussion of that definition.

Before providing a general definition of risk, several assumptions are made to simplify the exposition. All investors are the same and consume at only two dates in the future, date 1 and 2, respectively. It is assumed that the utility function is such that a globally accurate price index exists and that an indirect utility function of real consumption exists and is well-behaved. At time 0 a representative investor has wealth W_0 . Given his information at time 0, an investor can choose a portfolio to hold until date 1 and can choose actions he will undertake at time 1, given the realization of a set of random variables, and actions he will undertake at time 2, given the realization of a

set of random variables. It is assumed that an investor can trade assets or goods only at times 0, 1 and 2. Let H_0^i be the expected lifetime utility of an investor if he chooses a particular feasible program i at time 0. A feasible program satisfies the budget constraint of an investor at each date. Note that an investor's feasible program can be viewed as a portfolio plus a set of functions. Each function gives a set of actions which will be undertaken for given values of variables which, as of date 0, are random. We can choose to write

$$J_0 = \max_i \{H_0^i\}.$$

In other words, J_0 is the highest possible value of an investor's expected utility function of lifetime consumption at time 0.¹⁴ In an analogous way, we can define J_1 and J_2 . For instance, J_1 is the highest possible value of an investor's expected intertemporal utility function at date 1, before the investor has spent anything at that date. At time 0, J_1 has a distribution, with an expected value J_1 and a variance $\text{var}(J_1)$. An investor, who does not want to take any risks, wants $\text{var}(J_1)$ to be equal to zero. If the higher moments of the distribution of J_1 can be neglected, a natural measure of the riskiness of a security is its contribution to the variance of J_1 . A security whose price at date $t+1$ is positively correlated with J_1 — as viewed from date t — is more likely to have high payoffs when J_1 is high than when J_1 is low. If an investor sells one dollar of an asset uncorrelated with J_1 and buys one dollar of a security which has positive covariance with J_1 , the variance of J_1 increases, since the investor will be richer in states of the world when J_1 would have been high anyway. For the investor to buy the security whose price at date $t+1$ is positively correlated with J_1 , he must receive a risk premium which is proportional to the contribution of the security to $\text{var}(J_1)$.

The measure of risk just introduced is not observable. There exists a way to obtain an equivalent measure of risk which is observable. Let C_1^i be real consumption at date 1 if state of the world i occurs. If j and i are two states of the world, then it must be true that if $J_1^j > J_1^i$, it is the case that $C_1^j > C_1^i$. This means that if lifetime expected utility is unexpectedly high at date 1, then real consumption is also unexpectedly high. An increase in lifetime expected utility implies a fall in the marginal utility obtained from consuming a marginal (real) dollar, which for a state independent utility function of real consumption takes place through an increase in real consumption, i.e. C_1 . It follows that a risky asset will have a return which incorporates a positive (negative) risk premium if it is positively (negatively) correlated with C_1 . In the limit, when state variables and security prices follow Itô processes,

¹⁴ J is, in dynamic programming language, the optimal value function associated with the investor's problem.

unanticipated changes in the rate of consumption are perfectly correlated with changes in the function J .¹⁵ In this case, the risk premium is proportional to the covariance between the return of a risky asset and changes in the rate of consumption.¹⁶ It can be shown, with the use of stochastic calculus, that all investors need not be the same for this result to obtain and that they can consume different goods across countries.¹⁷ Moreover, it turns out that in continuous-time no assumption needs to be made about the utility function of investors, except that it is continuous, concave, and satisfies the von Neumann–Morgenstern axioms.

In a two-period framework, with the assumptions of this section, the measure of the risk of a security used in section 2 would provide the same ranking of securities as the measure just introduced. However, in a multi-period framework, an investor will always choose his consumption next period by taking into account the new information available about state variables, whereas the real wealth of the investor will not reflect that new information. The investor's real consumption will, *ceteris paribus*, increase if there is an unanticipated increase in wealth, but it will also increase if favorable changes occur in the investment opportunity set of investors.

Applying the measure of the riskiness of a security just introduced to a forward contract yields the result that the higher the covariance of changes in the exchange rate with changes in the world real consumption rate, the lower the forward exchange rate, *ceteris paribus*. It immediately follows that if macroeconomic policies affect the exchange rate and real consumption, one would expect that the forward exchange rate contains a risk premium. However, in this framework a change in macroeconomic policy matters to the extent that it affects the *ex ante* distribution of real consumption rather than to the extent that it affects key macroeconomic variables, like the money stock or the government budget. For instance, if it is assumed that the monetary authority controls the money stock, a decision to reduce the variance of the money stock could have a considerable impact on the risk premium contained in the forward exchange rate; whereas, a once and for all change in the money stock might have only a negligible effect or no effect at all. Clearly, our discussion does not leave any special role for the net supply of 'outside' bonds. It is easy to construct an example of a world in which there are outside bonds but in which one would not expect those bonds to imply the existence of a risk premium. Suppose that all investors are alike and that whenever transactions are permitted, the government pays interest on

¹⁵Intertemporal models using stochastic calculus were introduced by Merton; see for instance Merton (1973). For an introduction to stochastic calculus see Fischer (1975).

¹⁶Breeden (1979) introduces the covariance of asset returns with the change in the rate of growth of consumption as a measure of risk. Stulz (1981a) has a model which uses this measure of risk to study the pricing of assets in open economies.

¹⁷See Stulz (1981a).

nominal bonds and simultaneously taxes investors equally for an amount equal to the interest payments. In such a world, nominal bonds have to be held — there is an 'outside' supply — but they will not affect the behavior of investors.

6. Concluding remarks

There is a growing literature on the question of whether or not the forward rate is an unbiased predictor of the future spot rate.¹⁸ In this paper we have shown that the answer to this question depends upon the definition of risk. The most simple definition of risk yields a straightforward formula for the risk premium incorporated in the forward exchange rate. Unfortunately, that definition of risk is inappropriate when macroeconomic policies have real effects. If macroeconomic policies have real effects, it may be optimal for investors to hedge against unanticipated changes in macroeconomic policies.

For two assets to be perfect substitutes, an investor has to be indifferent to a trade in which his holdings of one asset are exchanged for holdings of the same value of the other asset. Clearly, if there is a risk premium incorporated in the forward exchange rate, domestic and foreign bonds cannot be perfect substitutes. Whereas the concept of risk used in section 2 implied that all assets with the same real expected rate of return are perfect substitutes, the concept of risk used in the later sections does not have that implication. In general, the absence of a risk premium on two particular assets does not mean that those assets are perfect substitutes for each investor.

Finally, whereas the concept of risk used in section 2 implies that the risk premium incorporated in the forward rate is fairly stable, no such implication is obtained from the results of the later sections. Specifically, section 5 suggests that the risk premium is the same whenever the state of the world is the same.

¹⁸More and more empirical papers are being written on this question, but we are not interested in them here.

References

- Adler, M. and B. Dumas, 1976, Portfolio choice and the demand for forward exchange, *American Economic Review* 66, 332–339.
- Black, F., 1974, International capital market equilibrium with investment barriers, *Journal of Financial Economics* 1, 337–352.
- Breedon, D.T., 1979, An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7, 265–296.
- Constantinides, G.M., 1980, Admissible uncertainty in the intertemporal asset pricing model, *Journal of Financial Economics* 8, 71–86.
- Dornbusch, R., 1980, Exchange risk and the macroeconomics of exchange rate determination, mimeo.

- Fama, E.F. and A. Farber, 1979, Money, bonds and foreign exchange, *American Economic Review* 69, 639-649.
- Fischer, S., 1975, The demand for index-bonds, *Journal of Political Economy* 83, 509-534.
- Frankel, F., 1979, The diversifiability of exchange risk, *Journal of International Economics* 9, 379-393.
- Grauer, F.L.A., R.H. Litzenberger and R.E. Stehle, 1976, Sharing rules and equilibrium in an international capital market under uncertainty, *Journal of Financial Economics* 3, 233-256.
- Heckerman, D., 1973, On the effects of exchange risk, *Journal of International Economics* 3, 379-387.
- Hellwig, M., 1977, On the validity of the intertemporal capital asset pricing model. CORE Discussion Paper, no. 7744.
- Kouri, P.K., 1976, The determinants of the forward premium, Institute for International Economic Studies, Stockholm, mimeo.
- Kouri, P.K., 1977, International investment and interest rate linkages under flexible exchange rates, in: R.Z. Aliber, ed., *The political economy of monetary reform* (New York).
- Lintner, J., 1965, Security prices, risk, and maximal gains from diversification, *Journal of Finance* 20, 587-615.
- Merton, R.C., 1971, Optimum consumption and portfolio rules in a continuous-time model, *Journal of Economic Theory* 3, 373-413.
- Merton, R.C., 1973, An intertemporal capital asset pricing model, *Econometrica* 41, 867-887.
- Rothschild, M. and J.E. Stiglitz, 1970, Increasing risk. I: A definition, *Journal of Economic Theory* 2, 225-243.
- Sharpe, W.F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425-442.
- Siegel, J., 1972, Risk, interest rates and the forward exchange rate, *Quarterly Journal of Economics* 86, 303-309.
- Solnik, B.H., 1973, Equilibrium in an international capital market under uncertainty, *Journal of Economic Theory* 8, 506-524.
- Stulz, R., 1980, *Essays on international asset pricing*, Ph.D. dissertation, M.I.T.
- Stulz, R.M., 1981a, A model of international asset pricing, *Journal of Financial Economics* 9, 383-406.
- Stulz, R.M., 1981b, On the effects of barriers to international investment, *Journal of Finance* 36, 923-934.
- Wihlborg, C., 1978, *Currency risks in international financial markets*, Princeton Studies in International Finance, no. 44.