

# Asset Pricing and Expected Inflation

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## ABSTRACT

This paper provides an equilibrium model in which expected real returns on common stocks are negatively related to expected inflation and money growth. It is shown that the fall in real wealth associated with an increase in expected inflation decreases the real rate of interest and the expected real rate of return of the market portfolio. The expected real rate of return of the market portfolio falls less, for a given increase in expected inflation, when the increase in expected inflation is caused by an increase in money growth rather than by a worsening of the investment opportunity set. The model has empirical implications for the effect of a change in expected inflation on the cross-sectional distribution of asset returns and can help to understand why assets whose return covaries positively with expected inflation may have lower expected returns. The model also agrees with explanations advanced by Fama [5] and Geske and Roll [10] for the negative relation between stock returns and inflation.

THE NEGATIVE RELATION BETWEEN stock returns and (a) expected inflation, (b) changes in expected inflation, and (c) unexpected inflation, has been extensively documented.<sup>1</sup> Fama [5] brings forth a large amount of evidence to justify the hypothesis that an unexpected increase in the growth rate of real activity not only causes an increase in stock prices, but also a decrease in the price level and a decrease in inflation because of its impact on money demand.<sup>2</sup> Geske and Roll [10] strengthen this hypothesis by showing that a decrease in economic activity increases the Federal budget deficit and that part of the deficit is monetized. Their analysis implies that a decrease in economic activity brings about an increase in the expected growth rate of the money supply and, hence, a larger increase in inflation than if there were no negative relation between real activity and the growth rate of the money supply. While the work of Fama [5], Nelson [22], and Geske and Roll [10] provides a convincing explanation of the relation

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<sup>1</sup> See Fama and Schwert [7], Jaffee and Mandelker [12], Nelson [21], and Schwert [24]. Fama [5] and Geske and Roll [10] provide some evidence which shows that the negative relation between ex ante stock returns and expected inflation may be partly spurious. Hasbrouck [11] finds that the negative relation of stock returns with expected inflation does not hold when he takes into account an ex ante measure of variability of real activity. In the model developed here, however, it turns out that variability of real activity and expected inflation are negatively related.

<sup>2</sup> Note that Hasbrouck [11] and Pearce and Roley [23] find that Fama's [5] results are less strong when survey data is used. Other explanations have been advanced for the negative relation between stock returns and inflation. See, e.g., Feldstein [8] and Modigliani and Cohn [19].

between stock returns and changes in expected inflation and the price level, it does not really explain “the most puzzling result of all” (Schwert [24, p. 28]), which is that ex ante real returns on common stocks are negatively related to ex ante expected inflation. Indeed, one would expect information about future economic activity to be capitalized in stock prices and to have no effect on ex ante real stock returns, i.e., on how investors value real cash flows.

Day [4] and LeRoy [14] provide valuable analyses which build on Fama [5] and show through the money demand equation that a negative relation between expected inflation and the expected real cash flows from invested capital can occur in efficient capital markets. However, their models take the expected real cash flows from invested capital to be exogeneously given. Consequently, they are unable to explain why expected stock returns, as opposed to expected real cash flows, are negatively related to expected inflation. They convincingly show, however, that when money demand is positively related to real cash flows, a decrease in expected real cash flows increases expected inflation for constant expected money growth.

Fama and Gibbons [6] provide an hypothesis which suggests a negative relation between endogenously determined ex ante returns on risky assets and expected inflation. They argue that an increase in economic activity increases not only inflation but also desired capital expenditures and hence increases the demand for funds to be invested in production. In this case, expected real rates of return must increase to equate investment and saving when an improvement in economic activity increases desired investment more than saving. While Fama and Gibbons [6] provide a story which fits the facts, they mainly explain the relation between the real rate of interest and economic activity, as they do not analyze explicitly the relation between expected inflation and the cross-sectional distribution of ex ante returns of risky assets.

This paper presents an equilibrium model in which the endogeneously determined expected real rate of return of the market portfolio of risky assets and the real rate of interest are negatively related to the level of expected inflation.<sup>3</sup> In this model, an increase in expected inflation, irrespective of its origin, decreases the real wealth of the households because it increases the opportunity cost of real balances and hence decreases the households' holdings of real balances. To keep all the stock of capital invested in production, the households' desired holdings of nominal assets, i.e., default-free nominal bonds and cash, must fall by the same dollar amount as their desired holdings of real balances. This can only be achieved by a fall in the real rate of interest which makes investments in nominal assets less attractive relative to investments in production. With a constant real rate of interest, there would be excess demand for nominal assets, as the relative investment proportions of risky assets are not affected by a fall in real wealth. Because of decreasing absolute risk aversion, the decrease in real wealth caused by the increase in expected inflation leads households to choose a portfolio of investments in production with a lower mean and variance of return.

In this paper, the expected real rate of return of the market portfolio falls less if expected inflation increases because of an increase in expected money growth

<sup>3</sup> The negative relation between the real rate of interest and expected inflation has been widely documented. See Fama and Gibbons [6] for references.

than if it increases because of a worsening of the production investment opportunity set (defined as the joint distribution of the returns of investments in production technologies). This follows from the fact that the portfolio households hold after an increase in expected money growth cannot be held if, instead, the production investment opportunity set has become less favorable. We show that, when expected inflation increases because of a worsening of the investment opportunity set, the expected real rate of return on the market portfolio of risky assets may fall by more than the real rate of interest. While our analysis provides a theoretical foundation for a negative regression coefficient of stock returns on expected inflation, the absolute value of the estimate of this coefficient presented by Fama and Schwert [7] is probably too large to be explained by the effects discussed in this paper.

The paper proceeds as follows. We present the model in Section I. Section II shows the effect of an increase in expected inflation on the expected real rate of return of the market portfolio for a given production investment opportunity set. In Section III, we discuss the effect of a change in the production investment opportunity set on the expected excess return of the market portfolio. While Sections I through III present our results when households have a logarithmic utility function of consumption services, Section IV shows that when households have a coefficient of relative risk aversion which exceeds one, they try to hedge against unanticipated changes in expected inflation. This means that assets whose return is positively correlated with changes in expected inflation have a lower expected return than predicted by the security market line. Consequently, our model offers a possible explanation for the empirical findings of Chen, Roll, and Ross [2] and Loo [16] that expected returns on risky assets are negatively related to their covariance with expected inflation. Section IV also uses our model to discuss Fama's [5] money demand argument. Section V offers some concluding remarks.

## I. The Model

The economy studied in this paper resembles closely the economy studied in Cox, Ingersoll, and Ross [3], except that households find it useful to hold cash balances in equilibrium.<sup>4</sup> Only one commodity is produced. Each household can invest in  $n - 1$  constant returns to scale production technologies, a nominal default-free bond, and a real default-free bond. Financial assets are assumed to trade on perfect markets, i.e., each household takes prices as given, there are no taxes or transaction costs and no restrictions on short sales. There is no outside supply of bonds. As all households are assumed to be the same, expected returns must be such that no household chooses to hold bonds. There is a government which changes the money stock by purchases and sales of the commodity and is assumed to make no transfers to households.<sup>5</sup>

<sup>4</sup> Other models which incorporate money in the Cox, Ingersoll, and Ross [3] model in a similar way are those of Jones [13], Gertler and Grinols [9], Stulz [26] and Stulz and Wasserfallen [28].

<sup>5</sup> More precisely, if transfers are made, they do not affect the marginal utility of wealth of households. Such an assumption makes sense if there is a positive relation between the deficit and money growth as in the Geske and Roll [10] analysis. Tobin [29] provides some justification for

To complete the description of the investment opportunity set, we need to specify the distribution of the real rate of return of the various assets households can hold. The instantaneous return of an investment of  $k_i$  in the  $i^{\text{th}}$  production technology is given by:

$$dk_i = \mu_{k_i} k_i dt + \sigma_{k_i} k_i dz_{k_i} \quad i = 2, \dots, n \quad (1)$$

where  $\mu_{k_i}$  and  $\sigma_{k_i}$  are taken to be constant in this section and the next and  $dz_{k_i}$  is the increment of a standard Wiener process. The instantaneous nominal rate of return of the safe nominal bond,  $R$ , is a function of the expected rate of change of the price of money,  $\pi$ , and cannot be assumed to be constant. It will be more convenient to work with the price of money rather than with its inverse, the price level. Expected inflation is equal to minus the expected rate of change of the price of money plus its variance. By definition, the instantaneous real rate of return on a nominal bond is  $Rdt + d\pi/\pi$  per unit of time. While the dynamics for  $\pi$  are solved for explicitly in Section III, we assume here that  $\pi$  follows:

$$d\pi = \mu_\pi \pi dt + \sigma_\pi \pi dz_\pi \quad (2)$$

where  $\sigma_\pi$  is constant and  $\mu_\pi$  changes over time according to:

$$d\mu_\pi = \gamma \mu_\pi dt + \sigma_\gamma \mu_\pi dz_\gamma \quad (3)$$

where  $\gamma$  and  $\sigma_\gamma$  are assumed to be constant for simplicity. We assume that the government chooses the dynamics of the money supply so that  $\pi$  follows Equation (2),  $\mu_\pi$  follows Equation (3), and the price of money is always expected to fall (i.e., expected inflation is always positive).<sup>6</sup> The instantaneous real rate of return on the safe real bond is  $r$ , and it is a function of  $\mu_\pi$  like  $R$ . All households are assumed to have the same real wealth,  $w$ , which is the sum of the *per capita* stock of capital,  $k$ , and real balances,  $m$ . Each household is infinitely lived and maximizes the following expected utility function of life-time consumption:

$$E_t \int_t^\infty e^{-\rho\tau} [\alpha \ln c(\tau) + (1 - \alpha) \ln m(\tau)] d\tau \quad (4)$$

where  $c(\tau)$  is the household's consumption of the commodity, and  $m(\tau)$  is its holdings of real balances. One possible justification for including real balances in the utility function is that households hold real balances to reduce the nonpecuniary costs associated with their trips to the bank to get cash to shop for the commodity.<sup>7</sup>

Let  $n_1 w$  be a household's holdings of nominal assets and let  $n_i w > 0$  be a household's investment in the  $i^{\text{th}}$  technology. With this notation, the flow budget

modeling the changes in the money stock as we do here. Stulz [27] argues that similar results obtain in the presence of transfers provided that a one percent increase in consumption expenditures leads to a less than one percent increase in real balances.

<sup>6</sup> If expected inflation could become negative, we would have to deal with the cumbersome problem that the nominal rate of interest cannot be negative.

<sup>7</sup> See McCallum [17] for a spirited defense of the practice of putting real balances in the utility function.

constraint of the household is given by:

$$dw = n_1 \left( Rdt + \frac{d\pi}{\pi} - rdt \right) w + \sum_{i=2}^n n_i \left( \frac{dk_i}{k_i} - rdt \right) w + rwdt - cdt - Rmdt. \quad (5)$$

The household solves for its portfolio by maximizing (4) subject to (5). Let  $\mathbf{n}$  be the  $n \times 1$  vector of shares of real wealth invested in risky assets. Solving for  $\mathbf{n}$ , we get:

$$\mathbf{n} = \mathbf{V}^{-1}(\boldsymbol{\mu} - r \cdot \mathbf{1}) \quad (6)$$

where  $\mathbf{V}^{-1}$  is the inverse of the  $n \times n$  variance-covariance matrix  $\mathbf{V}$  of real returns on investments,  $\boldsymbol{\mu}$  is the  $n \times 1$  vector of expected real returns on risky assets, and  $\mathbf{1}$  is a  $n \times 1$  vector of ones. The portfolio held by households is the market portfolio of the capital asset pricing model of Sharpe [25] and Lintner [15], except that here it is endogenously determined.

The solution of the portfolio selection problem yields the sum of the household's investments in real balances and nominal bonds. To obtain the household's real balances, one must use the properties of the utility function given by Equation (4). An infinitely lived household which maximizes (4) spends  $\rho w dt$  per unit of time on consumption services. As the expenditure share of real balances is  $(1 - \alpha)$ , this household spends  $(1 - \alpha)\rho w dt$  per unit of time to acquire the services of money. Hence, this reasoning implies that:

$$m = [(1 - \alpha)\rho w]/R. \quad (7)$$

In equilibrium, households hold no bonds, so that  $n_1 w = m$ .

It immediately follows from the solution of the household's portfolio choice problem that, as in the logarithmic utility version of the Cox, Ingersoll, and Ross [3] model, the expected real returns must satisfy the following equation:

$$\mu_i - r = \sigma_{i,w} \quad (8)$$

where  $\sigma_{i,w}$  is the instantaneous covariance between the real rate of return of the  $i^{\text{th}}$  risky asset and the rate of growth of real wealth. However, whereas in Cox, Ingersoll, and Ross [3], the distribution of the rate of growth of real wealth depends only on the investment opportunity set and the constraint that all wealth is invested in production, here the distribution of the rate of growth of real wealth must be such that the available stock of capital is wholly invested in production and that real balances held satisfy the condition given by Equation (7).

## II. Expected Returns and Expected Inflation

In this section, we study the effect of an increase in expected inflation under the assumption that the production investment opportunity set is constant. To simplify the analysis, we first assume that unanticipated changes in the price of

money are uncorrelated with the returns of each investment in production processes. In this case, the expected real rate of return of the market portfolio of investments in production is:

$$\mu_k = \frac{\boldsymbol{\mu}'_e \mathbf{V}_e^{-1} (\boldsymbol{\mu}_e - r \cdot \mathbf{1}_e)}{\mathbf{1}'_e \mathbf{V}_e^{-1} (\boldsymbol{\mu}_e - r \cdot \mathbf{1}_e)} \quad (9)$$

where the subscript  $e$  denotes the fact that the vectors and the matrix have been multiplied by an  $n \times n$  matrix,  $\mathbf{H}$ , which is the identity matrix except for zeros everywhere in the first row. Inspection of Equation (9) shows that a change in expected inflation can affect  $\mu_k$  only through its effect on the real rate of interest,  $r$ . Consequently, we first analyze the effect of an increase in expected inflation on the real rate of interest.

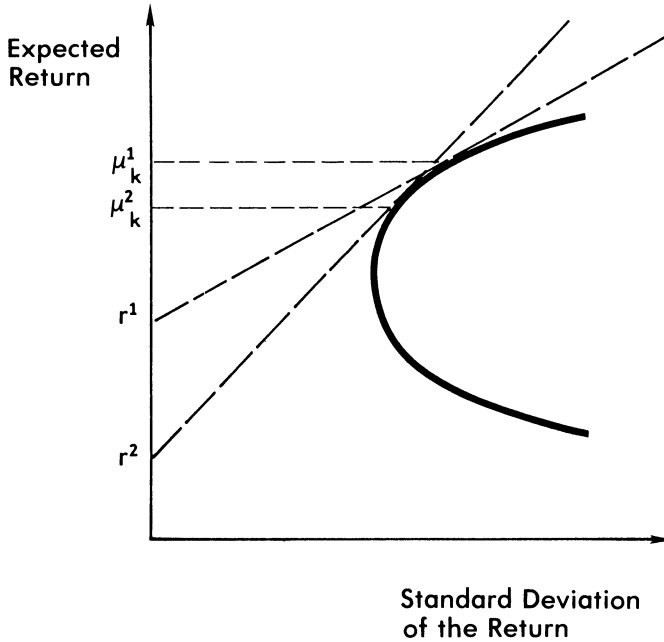
For a given real rate of interest and given holdings of real balances, the asset pricing equation derived in Section I, i.e., Equation (8), implies that an increase in expected inflation increases the nominal rate of interest,  $R$ , by the same amount. The money demand equation, i.e., Equation (7), shows that, for given consumption of the commodity, an increase in the nominal rate of interest decreases the holdings of real balances and hence decreases the price of money. A fall in real balances decreases real wealth, so that the nominal rate of interest is negatively related to real wealth. With constant relative risk aversion and a constant investment opportunity set, households invest a constant fraction of their wealth in production processes. Hence, for a constant real rate of interest, a fall in real wealth decreases the households' investment in production processes, which implies that part of the stock of capital is no longer invested. Instead, households want to invest part of their wealth in nominal bonds which are in zero net supply to keep the fraction of their wealth invested in nominal assets constant, as their holdings of real balances fell proportionately more than real wealth. To ensure that the whole stock of capital is invested in production, the fall in real wealth must, therefore, be accompanied by a fall in the real rate of interest<sup>8</sup> which makes investments in production more attractive relative to investments in nominal bonds. We can now state the first result of this paper:

**THEOREM 1.** *When the expected real rate of return on the market portfolio of investments in production is given by Equation (9), the instantaneous covariance of changes in the expected real rate of return of the market portfolio with changes in the real rate of interest is positive.*

*Proof:* See Appendix.

This result can be understood easily if one uses a familiar geometric tool. Figure 1 reproduces the efficient frontier of investments in production. For a real rate of interest,  $r^1$ , the market portfolio has an expected real rate of return,  $\mu_k^1$ . An increase in expected inflation makes households poorer, so that they no longer

<sup>8</sup> This fall in the real rate of interest also takes place, but for different reasons, in the models of Tobin [29] and Mundell [20]. In Mundell [20] the real rate of interest falls to equate savings and investment given *ad hoc* saving and investment functions, while in Tobin [29] the real rate of interest falls in *steady-state* because the marginal productivity of capital is lower as higher expected inflation leads to a higher steady-state capital stock.



**Figure 1.** A fall in the real rate of interest from  $r^1$  to  $r^2$  changes the tangency portfolio on the efficient frontier so that the expected return on the tangency portfolio falls from  $\mu_k^1$  to  $\mu_k^2$ .

want to invest the same number of units of the commodity in the portfolio with expected return,  $\mu_k^1$ . The real rate of interest falls to  $r^2$  to induce households to keep the number of units of the commodity invested in production constant. For the new real rate of interest,  $r^2$ , and the new risk premium required by households, the tangency portfolio has an expected real rate of return equal to  $\mu_k^2$ , such that  $\mu_k^2 < \mu_k^1$ . We can now state our second result:

**COROLLARY 1.** *An unanticipated increase in expected inflation decreases the expected real rate of return of the market portfolio and increases the price of production risk given by  $(\mu_k - r)/\sigma_k$ .*

Note that the price of production risk would correspond, in this model, to the empirical measure of the Sharpe-Lintner price of risk.<sup>9</sup> An increase in the price of production risk implies that the empirical Sharpe-Lintner security market line becomes steeper. Hence, the expected real rate of return of low risk assets is affected more by a change in expected inflation than the expected real rate of return on high risk assets. In fact, a security which has a high enough beta with respect to reference portfolios with expected real returns,  $\mu_k^1$  and  $\mu_k^2$ , has an expected return which increases with expected inflation. Notice however that while the two reference portfolios must have positively correlated real returns, the beta of most securities will change as expected inflation increases.

<sup>9</sup> The true measure of the Sharpe-Lintner price of risk would include holdings of real balances in the market portfolio.

To find out how our results are affected when we remove the assumption that unanticipated changes in the price of money,  $\pi$ , are uncorrelated with the returns of investments in production processes, one can use Equation (8) to obtain:

$$\mu_k = r + \sigma_{k,w} = r + \left(\frac{k}{w}\right) \sigma_k^2 + \left(\frac{m}{w}\right) \sigma_{k,\pi} \quad (10)$$

where  $\sigma_{k,\pi}$  is the instantaneous covariance between the rate of growth of the capital stock and the rate of growth of  $\pi$ . So far, we have assumed that  $\sigma_{k,\pi}$  is equal to zero. In this case, an increase in expected inflation increases  $k/w$ ; hence, for given  $\mu_k$  and  $\sigma_k^2$ ,  $r$  must fall when expected inflation increases. As long as this result holds, the value of  $\sigma_{k,\pi}$  does not affect our results. Notice that  $m/w$  is an increasing function of  $m$ . Hence, as long as  $\sigma_{k,\pi}$  is not too high, the results of this section hold. As, empirically,  $\sigma_{k,\pi}$  does not seem to exceed half of  $\sigma_k^2$ , our results hold for plausible values of  $\sigma_{k,\pi}$  (see Fama and Gibbons [6]). However, if  $\sigma_{k,\pi}$  were to exceed  $\sigma_k^2$ , our results would be reversed because, in this case, real balances would be very risky and a decrease in real balances held by households would decrease the variance of the rate of growth of real wealth. Notice also that the increase in the price of risk is a decreasing function of  $\sigma_{k,\pi}$  and that, when  $\sigma_{k,\pi} = \sigma_k^2$ , an increase in expected inflation does not affect the price of risk.

The results of this section provide a theoretical foundation for the existence of a negative relation between expected inflation and expected stock returns. However, Fama and Schwert [7] obtain an estimate for the regression coefficient of stock returns on the nominal rate of interest larger than five in absolute value. Such an estimate is much too large to be explained by the present analysis. It is obvious from Figure 1 that our analysis implies that the expected rate of return on the market portfolio has to fall by less than the real rate of interest. While some authors argue that the magnitude of the regression coefficient estimate obtained by Fama and Schwert [7] might be misleading,<sup>10</sup> most empirical studies have nevertheless presented results which indicate that an increase in expected inflation is likely to be associated with a fall in expected stock returns which is too large to be explained by the analysis presented so far. Therefore, we now turn to an extension of our analysis which can explain a larger absolute value for the regression coefficient of stock returns on expected inflation.

### III. Changes in the Production Investment Opportunity Set

In this section, we extend the model of Section I to allow for changes in the production investment opportunity set and investigate the relation between expected inflation and expected asset returns when the change in expected inflation is caused by a worsening of the production investment opportunity set. To model changes in the production investment opportunity set, we assume first that the expected real rate of return on an investment in the  $i^{\text{th}}$  technology is equal to  $\mu_{ki}x$  for all  $i$ 's, where  $x$  is a state variable which changes stochastically over time. The dynamics of  $x$  are left unspecified except that  $x$  always takes

<sup>10</sup> See, e.g., Geske and Roll [10].

positive values and follows a diffusion process. The variance of the real rate of return of investments in production technologies is left unchanged from Section I, so that the instantaneous output of the  $i^{\text{th}}$  technology is given by:

$$dk_i = \mu_{ki}xk_i dt + \sigma_{ki}k_i dz_{ki}, \quad i = 2, \dots, n \quad (11)$$

with Equation (11) instead of Equation (1), all the results of Section I still hold, except for the fact that all  $\mu_{ki}$ 's are multiplied by  $x$ .

We first consider the effect of an unanticipated fall in  $x$  assuming that the expected rate of growth of the money stock increases so that expected inflation is left unchanged. To simplify the discussion, we assume that unanticipated changes in the price of money are uncorrelated with output. In this case, Equation (9) can be rewritten as:

$$\mu_k = \frac{x\mu_e' V_e^{-1} [x\mu_e - r \cdot \mathbf{1}_e]}{\mathbf{1}_e' V_e^{-1} [x\mu_e - r \cdot \mathbf{1}_e]} \quad (12)$$

The effect of a fall in  $x$  on  $\mu_k$  and  $r$  is given by the following theorem:

**THEOREM 2.** *For a constant expected rate of change of the price of money, an unexpected fall in  $x$  decreases the expected real rate of return of the market portfolio of investments in production  $\mu_k$  and the real rate of interest,  $r$ . Furthermore,  $\mu_k$  falls by more than  $r$  and the price of production risk,  $(\mu_k - r)/\sigma_k$ , falls.*

*Proof:* See Appendix.

A fall in  $x$  decreases investment in production for a constant real rate of interest. Hence,  $r$  must fall to keep the stock of capital invested. However, the fall in  $x$  also shrinks the mean-variance efficient frontier of investments in production as the expected excess return of the  $i^{\text{th}}$  asset falls more than the expected excess return of the  $j^{\text{th}}$  asset whenever  $\mu_{ki} > \mu_{kj}$ . This fact induces households to hold a less risky portfolio, as such portfolios become relatively more advantageous. To keep the price of production risk unchanged,  $r$  would have to fall by more than the expected rate of return of the minimum variance portfolio of investments in production. Hence, with a constant price of production risk, the minimum variance portfolio becomes more attractive relative to risk-free investments. For households to keep investing a constant fraction of their wealth in production, the risk-free rate must fall to the same extent as the expected rate of return of the minimum variance portfolio. However, in this case,  $r$  falls by less than the expected rate of return of the market portfolio. If  $r$  falls by the same amount as the expected real rate of return of the minimum variance portfolio, the price of production risk must fall because the slope of the positively shaped segment of the efficient frontier of investments in production has fallen everywhere as a result of the decrease in  $x$ . This means that, as the investment opportunity set worsens, the absolute value of the difference between the expected returns of portfolios which differ in risk falls.

The analysis of the effects of an unanticipated fall in  $x$  is more complicated if the rate of growth of the money stock does not adjust to keep  $R$  constant. To see

this, suppose that the money stock dynamics are given by:

$$dM = \mu_M M dt + \sigma_M M dz_M \quad (13)$$

where  $\mu_M$  is fixed. In this case, using the money demand equation, i.e., Equation (7), equilibrium on the money market requires:

$$\pi M = \rho(1 - \alpha)k[R - \rho(1 - \alpha)]^{-1}. \quad (14)$$

Differentiating Equation (13) using Ito's Lemma and rearranging yields:

$$\begin{aligned} \frac{d\pi}{\pi} = & -\frac{dM}{M} + \frac{dk}{k} - \frac{dR}{R - \rho(1 - \alpha)} - \sigma_{k,M} dt \\ & + \sigma_M^2 dt + \sigma_{M,R} \left( \frac{R}{R - \rho(1 - \alpha)} \right) dt \end{aligned} \quad (15)$$

where  $dk$  is the rate of return on nonmonetary wealth,  $k$ . With a constant expected rate of growth of the money stock,  $R$  changes only because of changes in  $x$ . Hence, if  $x$  is given, inspection of Equation (15) shows that the expected rate of change of the price of money unambiguously increases with the expected rate of return of the market portfolio of investments in production. From the analysis of Section II, the fall in the expected growth rate of  $\pi$  increases the nominal rate of interest, decreases real balances, decreases both the real rate of interest and the expected rate of return on the market portfolio of risky assets, and increases the price of production risk. If the fall in the price of production risk caused by an increase in expected inflation is large, it could reverse the effect on the price of production risk of a worsening of the investment opportunity set for a given nominal rate of interest. However, this reversion can only happen if  $R$  is small, so that the fall in real balances following an increase in  $R$  is large. Hence, when expected inflation is high, an unexpected fall in  $x$  never increases the price of production risk with constant expected money growth. Our discussion at the end of the previous section implies that this result still holds if  $\sigma_{k,\pi}$  is positive or slightly negative. Hence, a proportional fall in the expected output rate of the risky technologies implies that the expected real rate of return on common stocks falls more with expected inflation than the real rate of interest. However, it follows from Equation (15) that, in this case, one would expect the regression coefficient of real stock returns on expected inflation to be minus one when expected inflation increases because of a worsening of the investment opportunity set. To explain the results of Fama and Schwert [7], one would need an unrealistically low income elasticity of money demand, so that an increase in the expected return of the market portfolio would be associated with a much lower increase in expected inflation. Notice, however, that the Geske and Roll [10] argument that a worsening in expected economic activity implies an increase in expected money growth would lead to a larger absolute value for the regression coefficient of stock returns on expected inflation when expected money growth is omitted from the regression. Unless the link between expected money growth and economic activity is extremely strong, one would not expect the analysis of this section to offer a satisfactory explanation for the results of Fama and Schwert [7].

In contrast to a proportional change in the expected rates of return of the technologies, an absolute change can never decrease the price of production risk. To see this, let the expected returns on investments in technologies be  $\mu_e + x\mathbf{1}_e$ . In this case, a fall in  $x$  implies a fall in the expected return of all technologies of equal magnitude. However, in this case, a fall in  $r$  of equal magnitude is enough to re-establish equilibrium on the market of investments in production. This means that expected excess returns on investments in production are unaffected. Hence, the expected return on the market portfolio falls to the same extent as the real rate of interest if  $R$  is maintained constant. In this case, because of the results of Section II and of the fact that expected inflation is inversely related to the expected real rate of return of the market portfolio, the real rate of interest falls by more than the expected real rate of return of the market portfolio of investments in production. Hence, the price of risk increases with constant expected money growth while the expected return of all technologies decreases by an equal amount. This effect does not obtain, however, if real balances are held constant. Finally, if  $\sigma_{k,\pi}$  differs from zero, this results holds provided that  $\sigma_{k,\pi}$  is no larger than  $\sigma_k^2$  and that the change in the production investment opportunity set has a negligible effect on  $\sigma_{k,\pi}$  for a constant,  $R$ .

It is useful to point out that a worsening of the investment opportunity set decreases the expected rate of return of the market portfolio by more than an increase in expected money growth. This follows from the fact that, in the case of a worsening of the investment opportunity set, the expected rate of return of the market portfolio falls even if expected inflation is maintained constant through a decrease in expected money growth.

#### IV. Extensions

In this section, we first show that if the coefficients of relative risk aversion of the representative investor exceeds one, assets whose return is positively correlated with changes in expected inflation have a lower expected return. We then show that the present model can also explain the negative correlation between stock returns and unexpected inflation.

##### A. Expected Inflation and Risk Premia

The work of Chen, Roll, and Ross [2] and Loo [16] implies that, empirically, assets whose return is positively correlated with changes in expected inflation have lower expected returns. This result can be understood within the present model if households maximize:

$$E_t \int_t^\infty e^{-\rho\tau} \frac{1}{\gamma} [c(\tau)^\alpha m(\tau)^{1-\alpha}]^\gamma d\tau.$$

If  $\gamma$  is negative, the coefficient of relative risk aversion of households exceeds one, so that households are more risk-averse than they were allowed to be earlier in this paper. Our earlier results do not depend on the precise value of  $\gamma$  as long as households are risk-averse. In this model, an increase in expected inflation brings about an increase in  $R$ , a fall in  $r$ , and a fall in the expected real rate of

return on the market portfolio of investments in production. An unanticipated increase in  $R$  is equivalent to an unanticipated increase in the cost-of-living of households because it increases the cost of using a constant quantity of services of real balances. This means that an unanticipated increase in expected inflation brings a once-and-for-all decrease in the households' lifetime expected utility. Households with a coefficient of relative risk aversion which exceeds one will want to hedge against unanticipated changes in expected inflation and hence will require a lower expected return on stocks whose return is positively correlated with changes in expected inflation.<sup>11</sup>

### B. Other Correlations

Fama [5] shows that, under some circumstances, money demand theory can explain the negative correlation between unexpected inflation and stock returns. His argument is that an unexpected fall in output implies a fall in stock prices as well as a fall in money demand. For a constant money stock, the fall in money demand implies a fall in the price level. LeRoy [14] and Day [4] present equilibrium models in which a negative correlation between inflation and stock returns occurs. Such a result can be obtained with the model developed in this paper. To construct an example in which stock returns are negatively correlated with unexpected inflation, suppose that the money stock *per capita*,  $M$ , follows the process assumed in Equation (13), but now  $\mu_M$  and  $\sigma_M$  can vary over time. In this case, assuming that households have logarithmic utility and using the equation for the equilibrium on the money market given in Section III, the instantaneous covariance between output and the rate of change of the price of money is:

$$\sigma_{k,\pi} = -\sigma_{k,M} + \sigma_k^2 - \sigma_{k,R}[R/(R - \rho(1 - \alpha))] \quad (16)$$

where  $\sigma_{k,\pi}$  and  $\sigma_{k,R}$  are, respectively, the instantaneous covariance of the real rate of return of investments in production with money stock growth and with the rate of change of the nominal rate of interest. Unless  $\sigma_{k,M}$  and  $\sigma_{k,R}$  are positive and large,  $\sigma_{k,\pi}$  is positive as argued by Fama [5].

## V. Concluding Remarks

This paper offers a model which makes it possible to understand the negative relation between expected real returns on common stocks and expected inflation. As the model has rich empirical implications, it should be possible to test it against other possible explanations of this negative relation. While it would be interesting to find out how our results hold up in a more general model, it is reassuring, however, that these results agree with the apparently successful money demand explanation for the negative relation between stock returns and unexpected inflation.

<sup>11</sup> See Breeden [1] for an analysis of the conditions under which households hedge.

**Appendix**

Proof of Theorem 1

Define:

$$a = \mathbf{1}'_e \mathbf{V}_e^{-1} \mathbf{1}_e \tag{A1}$$

$$b = \boldsymbol{\mu}'_e \mathbf{V}_e^{-1} \boldsymbol{\mu}_e \tag{A2}$$

$$f = \boldsymbol{\mu}'_e \mathbf{V}_e^{-1} \mathbf{1}_e. \tag{A3}$$

Differentiating Equation (9) using Ito's lemma and multiplying by  $d\mu_\pi$  yields:

$$\text{Cov}(d\mu_k, d\mu_\pi) = \frac{ab - f^2}{d - ra} \text{Cov}(dr, d\mu_\pi). \tag{A4}$$

Merton [18] proves that  $ab > f^2$ . Hence,  $\text{Cov}(d\mu_k, d\mu_\pi)$  has the same sign as  $\text{Cov}(dr, d\mu_\pi)$ . Note now that capital market equilibrium requires that:

$$\mathbf{1}'_e \mathbf{n}_e = fw - raw = k. \tag{A5}$$

Solving (A5) for  $r$  yields:

$$r = \frac{fw - k}{aw}. \tag{A6}$$

Differentiating (A6) using Ito's lemma and multiplying by  $d\mu_\pi$  yields:

$$\text{Cov}(dr, d\mu_\pi) = \frac{k}{aw^2} \text{Cov}(d\mu_\pi, dm) \tag{A7}$$

where we used the assumption that  $\text{Cov}(dk, d\mu_\pi) = 0$ . As  $\text{Cov}(d\mu_\pi, dm) > 0$ ,  $\text{Cov}(dr, d\mu_\pi) > 0$ . This concludes the proof.

Proof of Theorem 2.

Let  $f^* = xf$ . Replace  $f$  by  $f^*$  in (A6) and differentiate  $r$  with respect to  $x$  to get:

$$\frac{dr}{dx} = \frac{f}{a} > 0. \tag{A8}$$

Differentiate (12) with respect to  $x$  and use (A1) to (A3) and (A8) to get:

$$\frac{d\mu_k}{dx} = \frac{2x(b/f) - r - x(f/a)}{(1/f)(fx - ar)} > 0 \tag{A9}$$

where  $b/f > r$ ,  $b/f > f/a$ , and  $fx > ar$  from Merton [18]. Note now that:

$$\frac{d\mu_k}{dx} - \frac{dr}{dx} = \frac{2x[(b/f) - (f/a)]}{(1/f)(fx - ar)} > 0. \tag{A10}$$

Finally, the price of production risk,  $PR$ , is:

$$PR = (\mu_k - r)/\sigma_k = (\mu_k - r)^{1/2} \quad (\text{A11})$$

which implies that  $d(PR)/dx > 0$ .

#### REFERENCES

1. D. T. Breeden. "Futures Markets and Commodity Options: Hedging and Optimality in Incomplete Markets." *Journal of Economic Theory* 32 (April 1984), 275-300.
2. N.-F. Chen, R. Roll, and S. A. Ross. "Economic Forces and the Stock Market: Testing the APT and Alternative Asset Pricing Theories." Unpublished Working Paper, Center for Research in Security Prices, Chicago, IL, December 1983.
3. J. Cox, J. Ingersoll, and S. A. Ross. "An Intertemporal General Equilibrium Model of Asset Prices." *Econometrica* 53 (March 1985), 363-84.
4. T. E. Day. "Real Stock Returns and Inflation." *Journal of Finance* 39 (June 1984), 493-502.
5. E. F. Fama. "Stock Returns, Real Activity, Inflation, and Money." *American Economic Review* 71 (September 1981), 545-65.
6. ——— and M. R. Gibbons. "Inflation, Real Returns and Capital Investment." *Journal of Monetary Economics* 9 (May 1982), 297-324.
7. ——— and G. W. Schwert. "Asset Returns and Inflation." *Journal of Financial Economics* 5 (November 1977), 115-46.
8. M. Feldstein. "Inflation and the Stock Market." *American Economic Review* 70 (December 1980), 837-47.
9. M. Gertler and E. Grinols. "Monetary Randomness and Investment." *Journal of Monetary Economics* (September 1982), 239-58.
10. R. Geske and R. Roll. "The Fiscal and Monetary Linkages between Stock Returns and Inflation." *Journal of Finance* 38 (March 1983), 1-33.
11. J. Hasbrouck. "Stock Returns and Economic Activity: The Survey Evidence." *Journal of Finance* 39 (December 1984), 1293-1311.
12. J. F. Jaffe and G. Mandelker. "The 'Fisher Effect' for Risky Assets: An Empirical Investigation." *Journal of Finance* 31 (May 1976), 447-58.
13. E. P. Jones. "Intertemporal Financial and Monetary Equilibrium." Unpublished Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge, MA, 1980.
14. S. LeRoy. "Nominal Prices and Interest Rates in General Equilibrium: Endowment Shocks." *Journal of Business* 57 (April 1984), 197-214.
15. J. Lintner. "The Valuation of Risky Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budget." *Review of Economics and Statistics* 47 (February 1965), 13-37.
16. C.-H. Loo. "An Empirical Investigation of the Intertemporal Capital Asset Pricing Model under Expected Inflation." Unpublished Ph.D. Thesis, Ohio State University, Columbus, OH, February 1984.
17. B. T. McCallum. "The Role of Overlapping-Generations Models in Monetary Policy." *Carnegie-Rochester Conference on Public Policy* 18 (Spring 1983), 9-44.
18. R. Merton. "An Analytic Derivation of the Efficient Frontier." *Journal of Financial and Quantitative Analysis* 7 (September 1972), 185-72.
19. F. Modigliani and R. Cohn. "Inflation, Rational Valuation, and the Market." *Financial Analysts Journal* 35 (March/April 1979), 22-44.
20. R. A. Mundell. "Inflation and Real Interest." *Journal of Political Economy* 71 (June 1963), 280-83.
21. C. R. Nelson. "Inflation and Rates of Return on Common Stocks." *Journal of Finance* 31 (May 1976), 471-83.
22. ———. "Recursive Structure in U.S. Income, Prices and Output." *Journal of Political Economy* 87 (December 1979), 1307-27.
23. D. K. Pearce and V. V. Roley. "Stock Prices and Economic News." *Journal of Business* 58 (January 1985), 49-68.

24. G. W. Schwert. "The Adjustment of Stock Prices to Information about Inflation." *Journal of Finance* 36 (March 1981), 15-29.
25. W. F. Sharpe. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." *Journal of Finance* 19 (September 1964), 425-42.
26. R. M. Stulz. "Currency Preferences, Purchasing Power Risks and the Determination of Exchange Rates in an Optimizing Model." *Journal of Money, Credit and Banking* 16 (August 1984), 302-16.
27. ———. "Expected Inflation and Investment." Work in progress, 1985.
28. ——— and W. Wasserfallen. "Macroeconomic Time-Series, Business Cycles and Macroeconomic Policies." *Carnegie-Rochester Conference on Public Policy* 22 (Spring 1985), 9-54.
29. J. Tobin. "Money and Economic Growth." *Econometrica* 33 (October 1965), 671-84.