Default Risk, Idiosyncratic Coskewness and Equity Returns *

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Abstract

In this paper, we intend to explain an empirical finding that distressed stocks delivered anomalously low returns. We show that in a model with heterogeneous investors where idiosyncratic skewness is priced, the expected return of risky assets depends on idiosyncratic coskewness betas, which measure the covariance between idiosyncratic variance and the market return. We find that there is a negative (positive) relation between idiosyncratic skewness and equity returns when idiosyncratic coskewness betas are positive (negative). We construct two idiosyncratic coskewness factors to capture market-wide effect of idiosyncratic coskewness betas. When we control for these two idiosyncratic coskewness factors, the return difference for distress-sorted portfolios becomes insignificant. High stressed firm earn low returns because high stressed firms have high idiosyncratic coskewness betas when idiosyncratic coskewness betas are positive, and low idiosyncratic coskewness betas when idiosyncratic coskewness betas are negative.

1 Introduction

There have been considerable interest in understanding the relation between default-risk and equity returns in the finance literature. For example, Chan and Chen (1991) contend that the size effect is related to the presence of stressed firms in small stock portfolios. Similarly, Fama and French (1993) argue that a firm with high book-to-market ratio is relatively stressed. Vassalou and Xing (2004) use Merton (1974) model to compute default measures for individual firms and assess the effect of default risk on equity returns. They find that the size effect is a default effect, and this is also largely true for the book-to-market effect. However, recent empirical studies (Dichev (1998), Campbell, Hilscher, and Szilagyi (2008)) document a negative relationship between default risk and realized stock returns. In addition, Campbell, Hilscher, and Szilagyi (2008) find that correcting for risk using the standard risk factors worsens the anomaly.

In the credit risk literature, An empirical study by Zhang, Zhou, and Zhu (2008) finds that the volatility and jump risks of individual firms identified from high-frequency equity prices can predict about 50% of the variation in default risk, measured by credit default swap spreads. The jump risk alone forecasts about 20%. Since jumps affect higher moments of equity returns, their findings suggest a relationship between default risk and higher moments of equity returns. In addition, numerous papers (see Odean (1999), and Polkovnichenko (2005)) have documented that investors commonly do not hold well diversified portfolios. Given the lack of diversification in investor holdings, investors will care about the level of idiosyncratic skewness in their portfolio returns. Barberis and Huang (2008) and Mitton and Vorkink (2008) provide theoretical models to show that idiosyncratic skewness is priced in equilibrium. More recently, Boyer, Mitton, and Vorkink (2009) documents an empirical evidence that stocks with high expected idiosyncratic skewness have on average low returns. Because lagged skewness alone does not adequately forecast skewness, they regress time $t+1$ idiosyncratic skewness on time t predictor variables, and use the expected skewness as their measure of expected idiosyncratic skewness.

In this paper, we intend to examine the relationship between expected equity returns and another measure of idiosyncratic higher moments (hereafter idiosyncratic coskewness), and provide a possible explanation to the seemingly anomalous finding that high stressed firms earn low equity returns. There are several contributions of this paper.

First, we show that in a model in which there are heterogeneous investors who may care about the skewness of their portfolios, the expected return of risky assets depends on idiosyncratic coskewness betas, which measures the covariance between idiosyncratic variance and the market return. In addition, there is a negative (positive) relationship between idiosyncratic skewness and equity returns when the idiosyncratic coskewness beta is positive (negative).

Second, we test this prediction using equity returns. When estimated, idiosyncratic coskewness betas are positive, there is a negative relationship between excess returns and idiosyncratic coskewness betas. When estimated idiosyncratic coskewness betas are negative, the relationship becomes positive. In addition, when we control for risk using the market factor, the Fama-French three factors, and the Carhart four factors, the relationship between excess returns and idiosyncratic coskewness betas becomes stronger. In other words, the standard risk factors cannot explain why portfolios with low idiosyncratic coskewness betas earn high excess returns when idiosyncratic coskewness betas are positive, and why portfolios with high idiosyncratic coskewness betas earn high excess returns when idiosyncratic coskewness betas are negative. We form two long-short portfolios, which long the portfolio with the lowest idiosyncratic coskewness beta and short the portfolio with the highest idiosyncratic coskewness beta for both groups with positive and negative idiosyncratic coskewness betas, to capture the systematic variation in excess portfolio returns sorted by idiosyncratic coskewness betas. We call them idiosyncratic coskewness factors, $ICSK_1$ for the groups with positive idiosyncratic coskewness betas, and $ICSK_2$ for the groups with negative idiosyncratic coskewness betas. The average monthly excess returns for $ICSK_1$ and $ICSK_2$ over the sample period January 1971 to December 2006 are 0.61% ($t = 1.76$) and -0.76% ($t = 2.16$)

respectively.

Third, we show that the two idiosyncratic coskewness factors can explain the anomalous finding that high stressed firms earn low equity returns. As documented in Campbell, Hilscher, and Szilagyi (2008), the individual skewness is almost a monotonic function of their distress measure. We show that in an economy with investor with heterogeneous preferences, the equilibrium expected excess return depends on excess market return, idiosyncratic skewness, and idiosyncratic coskewness betas. We use Merton (1974) model to measure default risk for individual firms, and find the anomalous negative relation between default risk and equity returns. When we regress distress-sorted portfolio returns on the two idiosyncratic coskewness factors $ICSK_1$ and $ICSK_2$, we find that factor loadings on $ICSK_1$ are generally declining with distress measures increasing, and factor loadings on $ICSK_2$ are generally increasing with distress measures. The two idiosyncratic coskewness factors reduce the monthly excess return of a long-short portfolio holding the portfolio with the lowest distress measure and shorting the portfolio with the highest distress measure from 1.42% to 0.65%. Including other standard risk factors, such as the market, size, value, and momentum factors, will not significantly alter the factors loadings on the two idiosyncratic factors and the alpha of the long-short portfolio.

Our paper is one of several recent papers that intend to explain the anomalous finding that high stressed firms earn low equity returns. Garlappi and Yan (2008) show that in a model that considers the bargaining game between equity-holders and debt-holders, equity returns depend on a measure of "shareholder advantage". When shareholder advantage is strong, the relation between default probability and equity returns is humped and downward sloping. When shareholder advantage is weak, the relation is upward sloping. Avramov, Chordia, Jostova, and Philipov (2007a) show that the negative relationship between credit risk and average equity returns exist only during credit rating downgrade periods and is attributable to low-rated firms that experience considerable negative returns during the 1-year period around downgrades. Given the high return volatility of high stressed portfolios, Chava and Purnanandam (2008) argue against using realized returns as a proxy for expected returns. They use implied cost of capital computed from analysts forecasts as a measure of ex-ante expected returns. They find a significant positive relation between default risk and realized returns during the pre-1980 period. The negative relation between default risk and equity returns is a surprise to investors during the post-1980 period. Chen and Zhang (2008) propose a new three-factor model consists of the market factor and common factors based on investment and returns on asset. Their model can explain many empirical findings in finance, such as the positive relations of average returns with short-term prior returns and earnings surprises as well as the negative relations of average returns with financial distress, net stock issues, and asset growth. High stressed firms usually experience considerable negative returns in the past. It is no surprise that their model can explain the negative relation between default risk and equity returns since one of their factors is based on past equity returns.

Our paper provides an alternative explanation for the negative relation between default risk and equity returns. We show that high stressed firms earn low equity returns because of the contribution of their idiosyncratic skewness. For firms with positive idiosyncratic coskewness betas, high stressed firms have low idiosyncratic coskewness betas. In contrast, for firms with negative idiosyncratic coskewness betas, high stressed firms have high idiosyncratic coskewness betas. These relations contribute to low equity returns for high stress firms because there is a negative relation between equity returns and idiosyncratic coskewness betas for firms with positive idiosyncratic coskewness betas, and there is a positive relation between equity returns and idiosyncratic coskewness betas for firms with negative idiosyncratic coskewness betas.

This paper is organized as follows. Section 2 describes our measure of default risk based on Merton's (1974) model and the anomalous negative relation between default risk and equity returns. Section 3 presents both theoretical and empirical relation between idiosyncratic coskewness betas and equity returns. Section 4 examines the relation between default risk and idiosyncratic coskewness factors. Section 5 concludes.

2 Measuring Default Probability

2.1 Merton's Model

In the default risk literature, there are two approaches to measure default risk, the reduced-form and structural approaches. A reduced-form model provides the maximum likelihood estimates of a firm's default probability based on the empirical frequency of default and its correlation with various firm characteristics. A structural model provides estimated default probability which is theoretically motivated by the classical option-pricing models (Merton (1974)). Traditional reduced-form models, such as Altman (1968) Z-score model and Ohlson (1980) O-score model, compute measures of bankruptcy by using accounting information in conditional logit models. Accounting models use information from firms' financial statement. The accounting information is about firms' past performance, rather than their future prospects. In contrast, structural models use the market value of equity to derive measures of default risk. Market prices reflect investors' expectations about firms' future performance. Therefore, they are better suited for measuring the probability that a firm may default in the future. In this paper, we use Merton's (1974) model to estimate the default probability of a firm. In Merton's model, the equity of a firm is viewed as a call option on the value of firm's assets. The firm will default when the value of the firm falls below a strike price, which is measured as the book value of firm's liabilities. The firm value is not observable and is assumed to follow a geometric Brownian motion of the form:

$$dV_A = \mu_A V_A dt + \sigma_A V_A dW, \tag{1}$$

where V_A is the value of firm's assets, with an instantaneous drift μ_A, and an instantaneous volatility σ_A. W is the standard Wiener process.

Let X_t denote the book value of firm's liabilities at time t, which has a maturity at time T. The value of equity is given by the Black and Scholes (1973) formula for call options:

$$V_E = V_A N(d_1) + X e^{-rT} N(d_2), \tag{2}$$

where

$$d_1 = \frac{\ln(V_A/X) + (r + \frac{1}{2}\sigma_A^2)T}{\sigma_A \sqrt{T}}, d_2 = d_1 - \sigma_A \sqrt{T}, \tag{3}$$

r is the risk-free interest rate, and N is the cumulative density function of the standard normal distribution.

Using daily equity data from the past 12 months, we adopt a maximum likelihood method developed by Duan (1998) to obtain an estimate of the volatility of firm value σ_A. Duan (1998) computes the likelihood function of equity returns by utilizing the conditional density of the unobservable firm value process. We repeat the estimation procedure at the end of every month, resulting in monthly estimates of the volatility σ_A. We always keep the estimation window to 12 months.

With a estimated σ_A, we can calculate daily values of V_A for the last 12 months, and then estimate the drift μ_A. At the end of every month t, the probability of default implied by Merton's model is given by:

$$P_{def,t} = N(\frac{\ln(V_{A,t}/X_t) + (\mu_A - \frac{1}{2}\sigma_A^2)T}{\sigma_A \sqrt{T}}). \tag{4}$$

The $P_{def,t}$ calculated from equation (4) does not correspond to the true default probability of a firm in large samples since we do not use data on actual defaults. However, we use our measures to study the relationship between default risk and equity returns. The difference between our measure of default probability and true default probability may not be important as long as our measure correctly ranks firms according to their true default probability.

2.2 Data

One important parameter in Merton's model is the strike price, i.e. the book value of debt. Most firms have both long-term and short-term debts. Following KMV, we calculate the book value of debt by using short-term debt plus half long-term debt. We use the COMPUSTAT annual files to obtain the firm's "Debt in One Year" and "Long-Term Debt" series for all firms. Since debt data was not available for many firms before 1970, the sample period in our study is January 1971 to December 2006. In addition, financial firms have very different capital structure than industrial firms. We exclude all financial firms (SIC codes: 6000–6999). We also exclude all utility firms (SIC codes: 4900–4999) because many utility firms were highly regulated during our sample period. We use only industrial firms (SIC codes: 1–3999 and 5000–5999) in this studies since they are more suitable for Merton's model. We obtain all industrial firms with data available simultaneously on both CRSP and COMPUSTAT databases.

We obtain the book value of debt from the COMPUSTAT annual files. To avoid the problem of delayed reporting, we lag the book value of debt by 3 months. This is to ensure that our default probability measure is based on all information available to investors at the time of calculation.

To compute default likelihood measure, we obtain daily equity values for firms from CRSP daily files, and the risk-free interest rate from the Fama-Bliss discount bond file. We use monthly observations of the 1-year Treasury bill rate and equity data for the past 12 months to calculate monthly default measures for all firms.

When a firm is in sever financial distress, its equity is not liquid with low prices. To minimize liquidity effects on equity returns, we eliminate stocks with prices less than $1 at the portfolio construction date, and stocks with less than 120 transactions in the past 12 months. In the end, we have 10,078 firms with more than 3.5 million monthly observations in the sample.

Figure 1 plots the average default probability for industrial firms during the sample period. The shaded areas represent the NBER recession periods. The graph shows that the average default probability varies greatly and it usually peaks during recessions.

2.3 Measuring Performance of Merton's Model

To test the performance of Merton's model in predicting bankruptcy and other distress in our sample, we construct two measures based on exchange delisting as proxies for bankruptcy. One is a narrower measure of distress, called bankruptcy delisting (delisting codes: 400, 572, 574). The other is a broader measure of distress, called performance delisting (delisting codes: 400, 550 to 585). The second measure includes delisting due to not only bankruptcy and liquidation but also insufficient number of market makers, insufficient capital, surplus, and/or equity, price too low, delinquent in filing, etc. All the delisting data are obtained from CRSP.

To evaluate the predictive ability of our default measure to capture default risk, we sort firms according to their estimated default probability based on past 12-month equity data. At the end of each month from January 1971 through December 2006, default probability is re-estimated using only historical data to avoid look-ahead bias. To pay great attention to the tail of default risk distribution, we follow Campbell et. al. and construct 10 portfolios contain stocks in percentiles 0–5, 5–10, 10–20, 20–40, 40–60, 60–80, 80–90, 90–95, 95–99, and 99–100 (P1 and P10 denote the portfolios with the lowest and the highest default probability respectively). In the following month, we then collect the number of bankruptcy and performance delistings for each portfolio. The summary results are reported in Table 1. The evidence shows that the default risk measure based on Merton's model is a good ex ante measure of probability of bankruptcy and other distress. The number of bankruptcy and performance delistings generally increases with default risk measures from Merton's model. During the sample period, 46 out of 60 delistings due to bankruptcy and liquidation, and 213 out of 443 delistings due to performance come from the two portfolios with highest default measures. These two portfolios contains the highest-risk 5% of stocks.

2.4 Equity Returns on Distressed Stocks

We now examine the relationship between the likelihood of default and equity returns. Fama and French (1996) and others have argued that financial stress may explain the size and value premiums in equity returns. Vassalou and Xing (2004) provide some supporting evidences. This conjecture, however, has proven difficult to reconcile with other empirical findings by Dichev (1998) and Campbell, Hilscher, and Szilagyi (2008), which show strong underperformance among the group of firms with the highest measures of financial distress.

We use the same method, i.e. Merton's model, to estimate default likelihood as Vassalou and Xing (2004). Unlike Vassalou and Xing (2004), we use only industrial firms, which are more suitable for Merton's model. We also minimize liquidity effects on equity returns by eliminating illiquid stocks. At the end of each month, we sort firms according to their default measures and construct 10 portfolios as discussed in the previous section. Because highly distressed firms are more likely to be delisted and disappear from the CRSP database, it is important to carefully compute equity returns for delisted firms. CRSP reports a delisting return for the final month of a firm's life when it is available. In this case, we use delisting returns to compute portfolio returns. When delisting returns are not available, we exclude those firms from portfolios. This assumes that those stocks are sold at the end of the month before delisting, which implies an upward bias to the returns for distressed-stock portfolios (Shumway (1997)).

Table 2 reports the summary statistics of equity returns on the ten distress-sorted portfolios. The average returns are declining in general with default measures increasing. The average return is 0.92% for the portfolio with the lowest default risk, and it is -0.51% for the portfolio with the highest default risk. The volatilities of returns are increasing with default measures. The standard deviation of returns is 4.48% for the portfolio with the lowest default risk, and it is 14.95% for the portfolio with the highest default risk. In addition, returns on portfolios with low default measures exhibit negative skewness, and returns on portfolio with high default measures exhibit positive skewness. The average size of firms in the ten portfolios is monotonically declining with default measures increasing. It suggests that controlling size risk factor will not explain the puzzling negative relation between equity returns and default risk.

A possible explanation for the negative relation between equity returns and default measures is that the default measure is just a proxy for other systematic risk factors. We test this hypothesis with regressions. Table 3 reports the regression results. Panel A reports the excess returns of ten distress-sorted portfolios and a long-short-portfolio that goes long the portfolio with the lowest default risk, and short the portfolio with the highest default risk. Panel A also reports the alphas in regressions of the portfolio excess returns on the CAMP factor, Fama-French three factors, and four factors proposed by Carhart (1997) that includes a momentum factor in addition to Fama-French three factors. The returns are reported in monthly percentage points, with Robust Newey-West t-statistics below in the parentheses. Panel B, C, and D report estimated factor loadings for excess returns on the CAMP factor, Fama-French three factors, and four factors in the Carhart (1997) model. Figure 2 plots the alphas from these regressions.

The average excess returns of the 10 stress-sorted portfolios reported in Table 3 are in general declining in the default risk measure. The average excess return for the lowest-risk 5% of stocks is positive at 0.43% per month, and the average excess return for the highest-risk 1% of stocks is negative at -0.99% per month. A long-short portfolio that goes long the safest 5% of stocks, and short the most distressed 1% of stocks has an average return of 1.42% per month with a standard deviation of 14%. It implies a Sharp ratio of 0.10.

There is also a significant pattern on the factor loadings reported in Table 3. The low risk portfolios in general have smaller market betas, negative loadings on the size factor *SMB*, and negative loadings on the value factor *HML*. On the contrary, the high risk portfolios in general have bigger market betas, positive loadings on the size factor *SMB*, and positive loadings on the value factor *HML*. The results reflect the fact that most distressed stocks are small stocks with high book-to-market ratios. It implies that correcting risk using market factor or Fama-French factors will not solve the anomaly but worsen it. In fact, the long-short portfolio longing the safest 5% of stocks, and shorting the most distressed 1% of stocks has a CAMP alpha of 1.94% per month with a t-statistic of 3.16. It has a Fama-French three-factor alpha of 2.76% per month with a t-statistic of 4.90. In addition, the Fama-French three-factor alphas for all portfolios beyond 40th percentile of the default risk distribution are negative and statistically significant.

Avramov, Chordia, Jostova, and Philipov (2007b) find a robust link between credit rating and momentum. They find that momentum profit exists only in low-grade firms. Distressed firms have negative momentum, which may explain their low average returns. When we correct for risk by using Carhart (1997) four-factor model including a momentum factor, the low risk portfolios in general have low and positive loadings on the momentum factor. The high risk portfolios have high and negative loadings on the momentum factor. After controlling for the momentum factor, we find that the alpha for the long-short portfolio is cut almost in half, from 2.76% per month to 1.38% per month, which is still statistically significant.

3 Idiosyncratic Coskewness and Equity Returns

3.1 Theory

Many empirical papers have documented that investors usually hold under-diversified portfolios with a small number of securities. One possible explanation is that investors care about idiosyncratic skewness in their portfolios. Barberis and Huang (2008) show that idiosyncratic skewness is priced in equilibrium under the assumption that investors have preferences based on cumulative prospect theory. Mitton and Vorkink (2008) demonstrate the same result under the assumption of heterogeneous preference for skewness. Campbell, Hilscher, and Szilagyi (2008) report average skewness of individual returns for distress-sorted portfolios in Table VI of their paper. The result shows that average skewness of individual returns are almost monotonically increasing with default risk. In other words, high default-risk stocks have high skewness returns. We intend to examine the relationship between equity returns and idiosyncratic skewness in a linear asset pricing model, and investigate empirically if investors' preference for idiosyncratic skewness can explain the anomaly that high distressed firms earn low returns.

In our model, we follow Mitton and Vorkink (2008) and assume that the universe of stocks consists of three assets and a risk-free asset. The return vector of the two securities is denoted as $R = [R_M, R_1]$. The covariance of asset returns is denoted Σ.

In our economy, we assume that they are two investors, a traditional investor and a "Lotto investor". The traditional investor utility can be approximated as a standard quadratic utility function over wealth

$$U(\mathcal{W}) \simeq E(\mathcal{W}) - \frac{1}{2\tau} Var(\mathcal{W}), \tag{5}$$

where $\mathcal{W}$ is the investor terminal wealth, $\tau > 0$ is the coefficient of risk aversion. Markowitz (1979) and Hlawitschka (1994) show that the quadratic utility is a reasonable approximation of standard expected utility functions. And it seems reasonable to assume that in the population traditional investor behaves as mean-variance investors. The "Lotto Investor" has the same preferences as the traditional investor over mean and variance, but also has preference for skewness

$$U(\mathcal{W}) = E(\mathcal{W}) - \frac{1}{2\tau} Var(\mathcal{W}) + \frac{1}{3\phi} Skew(\mathcal{W}), \tag{6}$$

where ϕ is the investor skewness preference. As shown in Cass and Stiglitz (1970), utilities (5) and (6) can lead, under certain restrictions, to equilibrium portfolio separation. As $\phi \longrightarrow \infty$, the Lotto investor utility approaches the traditional investor utility as in Markowitz (1959). It is insightful to notice that if all investors are lotto investors, then the model would be reduced to Kraus and Litzenberger (1976) coskewness model. Each investor maximize (5) and (6) subject to his budget constrain of the form

$$\mathcal{W}_i = \mathcal{W}_{0,i} R_f + \omega_i^T (R - R_f \mathbf{1}),$$

where R_f is the return on the risk-free asset, $R - R_f\mathbf{1}$ is the excess return, and ω_i is the asset demand. For the traditional (hereafter $\mathcal{T}$) investor, the First-order conditions of (5) gives

$$E(R - R_f\mathbf{1}) - \frac{1}{\tau}\Sigma\omega_{\mathcal{T}} = 0. \tag{7}$$

For Lotto investors (hereafter $\mathcal{L}$), the First-order conditions can be simplified to

$$E(R - R_f\mathbf{1}) - \frac{1}{\tau}\Sigma\omega_{\mathcal{L}} + \frac{1}{\phi}E(\omega_{\mathcal{L}}^T (R - ER)(R - ER)^T \omega_{\mathcal{L}})(R - ER) = 0. \tag{8}$$

Since the economy should generate the same expected excess return regardless of investor preferences, the expected excess returns in (7) and (8) have to be identical. This allows us to write the equilibrium expected excess return as

$$E(R - R_f\mathbf{1}) = \frac{1}{2\tau}\Sigma(\omega_{\mathcal{L}} + \omega_{\mathcal{T}}) - \frac{1}{2\phi}E(\omega_{\mathcal{L}}^{\mathcal{T}} (R - ER)(R - ER)^T \omega_{\mathcal{L}})(R - ER).$$

Notice that $\omega_{\mathcal{L}} + \omega_{\mathcal{T}}$ represents the aggregate demand in this economy, hence $(\omega_{\mathcal{L}} + \omega_{\mathcal{T}})^{\mathcal{T}} R$ can be treated as the return on the market portfolio. Finally, the equilibrium expected excess return on the risky asset is

$$E(R_1 - R_f) = \frac{1}{2\tau}Cov(R_1, R_M) - \frac{1}{2\phi}Cov((\omega_{\mathcal{L}}^{\mathcal{T}} (R - ER)(R - ER)^{\mathcal{T}} \omega_{\mathcal{L}}), R_1). \tag{9}$$

Equation (9) is not the coskewness model put forward in Harvey and Siddique (2000) nor the coskewness model of Kraus and Litzenberger (1976). Let $\gamma_{ISK} = E(R_1 - ER_1)^3$ denote the asset one idiosyncratic skewness. The expected excess return on the risky asset (9) is

$$E(R_1 - R_f) = \lambda_M \beta_1 + \lambda_{CSK} \beta_{CSK} + \lambda_{ICSK} \gamma_{ICSK} + \lambda_{ISK} \gamma_{ISK},$$

where

$$\beta_{CAMP} = \frac{Cov(R_1, R_M)}{Var[R_M]}, \quad \beta_{CSK} = \frac{Cov(R_1, (R_M - ER_M)^2)}{Var[R_M]}, \; \beta_{ICSK} = \frac{Cov((R_1 - ER_1)^2, R_M)}{Var[R_M]},$$

with

$$\lambda_M = \frac{1}{2\tau}\sigma_M^2, \; \lambda_{CSK} = -\frac{(\omega_{\mathcal{L},M})^2}{2\phi}\sigma_M^2, \; \lambda_{ICSK} = -\frac{\omega_{\mathcal{L},M}\omega_{\mathcal{L},1}}{\phi}\sigma_M^2, \; \lambda_{ISK} = -\frac{(\omega_{\mathcal{L},1})^2}{2\phi}.$$

The expected excess return depends on the standard CAMP beta, β_{CAMP}, the coskewness beta as defined in Harvey and Siddique (2000), β_{CSK}, the idiosyncratic coskewness beta, β_{ICSK}, and the expected idiosyncratic skewness as defined in Mitton and Vorkink (2008).

The impact of the standard CAMP beta, the coskewness beta, and the expected idiosyncratic skewness on asset returns is consistent with theoretical predictions since λ_M is positive, and λ_{CSK} and λ_{ISK} are negative. However, the impact of the idiosyncratic coskewness beta on asset returns is ambiguous because λ_{ICSK} can be positive and negative, depending on whether the lotto investor buy or sell the risk assets.

To investigate the relation between idiosyncratic coskewness betas and expected returns we consider two assets and form a portfolio of these two assets by changing the weight on these assets from -1 to 1. We then study the return difference between the portfolio with the highest idiosyncratic coskewness beta and the portfolio with the lowest idiosyncratic coskewness beta. To perform our analysis, we fix the returns of the two assets and their idiosyncratic coskewness betas. The top left graph in Figure 3 shows the relationship between the portfolio idiosyncratic coskewness beta and the expected return when the idiosyncratic coskewness betas for both assets are positive. SET1 contains the expected returns and idiosyncratic coskewness betas of the two assets. As shown in this graph, the difference in expected returns between the portfolio with the highest idiosyncratic coskewness beta and the portfolio with the lowest idiosyncratic coskewness beta is negative. We reach the same conclusion in the top left graph in Figure 4.

The bottom right graph in Figure 3 shows the relationship between idiosyncratic coskewness betas and expected returns when the idiosyncratic coskewness betas for both assets are negative. SET4 contains the expected returns and idiosyncratic coskewness betas of the two assets. As shown in this graph, the difference in expected returns between the portfolio with the highest idiosyncratic coskewness beta and the portfolio with the lowest idiosyncratic coskewness beta is positive. We reach the same conclusion in the bottom right graph in Figure 4.

The top right graph in Figure 3 shows the relationship between idiosyncratic coskewness betas and expected returns when asset 1 has negative idiosyncratic coskewness beta and asset 2 has positive idiosyncratic coskewness beta. The bottom left graph in Figure 3 shows the relationship between idiosyncratic coskewness betas and expected returns when asset 1 has positive idiosyncratic coskewness beta and asset 2 has negative idiosyncratic coskewness beta. As shown in these graphs, there is no a clear relationship between the portfolio idiosyncratic coskewness beta and its expected return. We reach the same conclusion in the top right and bottom left graphs in Figure 4.

3.2 Portfolio Returns Sorted by Idiosyncratic Coskewness

We use the entire CRSP equity data to investigate the relationship between equity returns and idiosyncratic coskewness betas. At the beginning of each month, we use past 12-month daily data on individual stock returns to compute idiosyncratic coskewness betas as defined the previous section. To reduce the liquidity effect on equity returns, we eliminate firms with no transaction days larger than 120. We also eliminate stocks with prices less than $1 at the end of a month. Since the effect of idiosyncratic skewness on expected equity returns might depend on the sign of idiosyncratic coskewness beta, $\gamma_{i,t}$, we divide firms into two groups according to the sign of $\gamma_{i,t}$. For each group, we then rank the stocks based on their past $\gamma_{i,t}$ and form ten value-weighted decile portfolios. Following the same method used to compute returns for distress-sorted portfolios, we compute value-weighted returns for idiosyncratic coskewness beta-sorted portfolios in each groups.

Tables 4 and 5 report the results for ten portfolios with positive idiosyncratic coskewness betas and ten portfolios with negative idiosyncratic coskewness betas respectively. Panel A reports average excess returns, in monthly percentage points, of idiosyncratic coskewness beta-sorted portfolios and the average return of a long-short-portfolio holding the portfolio with the lowest idiosyncratic coskewness beta and shorting the portfolio with the highest idiosyncratic coskewness beta. Panel A also reports alphas with respective to the CAMP, the Fama-French three-factor model, and the four-factor model proposed by Carhart (1997) that includes a momentum factor. Panel B reports estimated factor loadings in the four-factor model with adjusted R^2s. Figures 5 and 6 plot the alphas from regressions for ten positive portfolios with positive idiosyncratic coskewness betas and ten portfolios with negatively idiosyncratic coskewness betas respectively.

The average excess returns for the first nine portfolios with positive idiosyncratic coskewness betas are almost flat. The average excess return for the tenth portfolio, which has the highest idiosyncratic coskewness beta, is much lower than those for the other nine portfolios. The average return for the long-short-portfolio which goes long the portfolio with the lowest idiosyncratic coskewness beta and short the portfolio with the highest idiosyncratic coskewness beta is 0.81% with a t-statistic of 1.87. The results weakly support the prediction that excess returns decline with idiosyncratic coskewness betas rising when idiosyncratic coskewness betas are positive.

There is also interesting pattern in estimated factor loadings reported in Table 4. Portfolios with low idiosyncratic coskewness betas have low loadings on the market factor, negative loadings on the size factor *SMB*, and positive loadings on the value factor *HML*. Portfolios with high idiosyncratic coskewness betas have high loadings on the market factor, positive and high loadings on the size factor *SMB*, and negative loadings on the value factor *HML*. There is no clear pattern in the estimated factor loadings for the momentum factor *UMD*.

These factor loadings implies that when we correct risk using the market factor or the Fama-French three factors, we will not be able to explain why the portfolio with the highest idiosyncratic coskewness beta has such low excess returns comparing to the other nine portfolios. On the contrary, it will worsen the anomaly. In fact, alphas in the regressions with respect to the CAMP, the Fama-French three-factor model, and Carhart four-factor model are almost monotonic declining with idiosyncratic coskewness betas increasing. A long-short portfolio that holds the portfolio with the lowest idiosyncratic coskewness beta and shorts the portfolio with the highest idiosyncratic coskewness beta has a CAMP alpha of 1.21% with a t-statistic of 3.14; it has a Fama-French three-factor alpha of 1.12% with a t-statistic of 4.29; and it has a Carhart four-factor alpha of 0.98% with a t-statistic of 4.02. When we correct risk using the standard factors, we find stronger evidence to support the prediction that there is a negative relationship between excess returns and idiosyncratic coskewness betas when idiosyncratic coskewness betas are negative.

For the ten portfolios with negative idiosyncratic coskewness betas, the average excess returns reported in Table 5 are almost monotonically increasing with idiosyncratic coskewness betas. It is consistent with the prediction that there is a positive relationship between excess returns and idiosyncratic coskewness betas when idiosyncratic coskewness betas are negative. A long-short portfolio that holds the portfolio with the lowest idiosyncratic coskewness beta and shorts the portfolio with the highest coskewness beta has an excess return of -0.63% with a t-statistic of 2.00.

There is a clear pattern in estimated factor loadings for the market factor and the size factor *SMB* in the four-factor regression. Portfolios with low idiosyncratic coskewness betas have high loadings on the market factor and the size factor *SMB*. Portfolios with high idiosyncratic coskewness betas have low loadings on the market factor and the size factor

SMB. There is no clear pattern in estimated factor loadings for the value factor *HML* and the momentum factor *UMD*. These loading implies that when we cannot explain the return difference using the standard risky factors. In fact, controlling those factors increases return difference for portfolios sorted by idiosyncratic coskewness betas. The same long-short portfolio has a CAMP alpha of -0.88% with a t-statistic of 2.74; it has a Fama-French three-factor alpha of -0.85% with a t-statistic of 3.76; and it has a Carhart four-factor alpha of -0.61% with a t-statistic of 2.48.

In summary, the empirical results support that the relationship between equity returns and idiosyncratic coskewness betas is positive when idiosyncratic coskewness betas are negative, and negative when idiosyncratic coskewness betas are positive. In addition, we find that return difference between portfolios sorted by idiosyncratic coskewness betas, with either positive or negative values, cannot be explained by the standard risk factors, such as the market factor, the size factor, the value factor, and the momentum factor. In the next section, we will examine the relationship between default risk and idiosyncratic coskewness.

3.3 Idiosyncratic Coskewness Factors

We investigate two value-weighted hedge portfolios that capture the effect of idiosyncratic coskewness. As discussed in the previous section, at the beginning of each month, we use past 12 month daily equity returns to estimate idiosyncratic coskewness beta for each individual firm. We first divide firms into two groups according to the sign of estimated idiosyncratic coskewness betas, then we form value-weighted decile portfolios based estimated idiosyncratic coskewness betas for both groups. We compute the excess portfolio returns in the following month (i.e. post-ranking). We construct the long-short portfolio holding the portfolio with the lowest idiosyncratic coskewness beta and shorting the portfolio with the highest idiosyncratic coskewness beta. The long-short portfolio in the group with negative idiosyncratic coskewness beta is called $ICSK_1$, and the long-short portfolio in the group with positive idiosyncratic coskewness beta is called $ICSK_2$. We use $ICSK_1$ and $ICSK_2$ to proxy for idiosyncratic coskewness factors.

The average monthly excess returns for $ICSK_1$ and $ICSK_2$ are 0.81% and -0.63% respectively over the period January 1971 to December 2006. We reject the hypothesis that the mean excess return for factor $ICSK_2$ is zero at the 5 percent level of significance. But we cannot reject the same hypothesis for factor $ICSK_1$. A high factor loading on $ICSK_1$ should be associated with high expected excess returns. In contrast, for factor $ICSK_2$, a high factor loading should be associated with low expected excess returns.

To show the difference between idiosyncratic coskewness and idiosyncratic skewness, we also calculate returns for portfolios sorted by idiosyncratic skewness. Using the same CRSP data, at the beginning of each month, we use past 12-month daily data on individual stock returns to estimate an empirical CAMP model. We construct 10 decil portfolios according to the skewness of the residuals from the regression. We then compute value-weighted excess returns for the 10 idiosyncratic skewness-sorted portfolios. Table 6 shows that the unconditional excess returns are flat. The alphas are not statistically significant once we control the standard risk factors, such as Fama-French and Momentum factors. Table 6 confirms our theoretical finding our measure of idiosyncratic coskewness is different from the standard idiosyncratic skewness measure. We do not investigate the empirical relationship between expected idiosyncratic skewness and idiosyncratic coskewness[1].

3.4 Can Idiosyncratic Coskewness Factors Explain What Other Risk Factors Do Not?

The failures of the CAMP model often appear in specific groups of securities that are formed on size, book-to-market ratio and momentum. To understand how idiosyncratic coskewness factors enter asset pricing, we analyze the pricing errors from other asset pricing models such as the Fama-French three-factor model, and the four-factor model proposed by Carhart

[1]It will be interesting to investigate the relationship between the expected idiosyncratic co-skewness and expected idiosyncratic skewness measures.

(1997).

We carry out time-series regression of excess returns,

$$r_{i,t} = \alpha_i + \sum_{j=1}^{K} \widehat{\beta}_j f_{j,t} + e_{i,t}, \text{ for } i = 1, ..., N, t = 1, ..., T, \tag{10}$$

and jointly test whether the intercepts, α_i, are different from zero using the F-test of Gibbons, Ross, and Shanken (1989) where $F \sim (N, T - N - K)$. We test the Fama-French three factor model and Carhart four-factor model for industrial portfolios, decil portfolios sorted by size, book-to-market ration, and momentum, and decil portfolios sorted by idiosyncratic coskewness beta. The results are presented in Table 7. When we test 10 portfolios sorted by the book-to-market ratio, the inclusion of the two idiosyncratic coskewness factors reduces the F-statistics from 4.96 to 1.95 in the Fama-French model and from 3.39 to 1.31 in the Carhart model. Similar results are obtained for momentum-sorted portfolios and portfolios sorted by idiosyncratic coskewness beta. In all cases, the inclusion of the two idiosyncratic coskewness factors in either the Fama-French model or the Carhart model dramatically reduces the F-statistics. The results suggest that the two idiosyncratic coskewness factors can explain a significant part of the variation in returns even when factors based on size, book-to-market ratio, and momentum are added to the asset pricing model. In the next section, we will examine the relationship between default risk and idiosyncratic coskewness.

4 Default Risk and Idiosyncratic Coskewness

4.1 Explaining Equity Return Anomaly for Distressed Firms

We have demonstrated that the marginal contribution of expected idiosyncratic skewness to expected excess returns depends on idiosyncratic coskewness betas. For positive idiosyncratic coskewness betas, high idiosyncratic skewness should reduce expected excess returns. For negative idiosyncratic coskewness betas, high idiosyncratic skewness should increase expected excess returns. One possible explanation for low equity returns on high distress firms is that high distress firms have high idiosyncratic skewness when their idiosyncratic coskewness betas are positive, or high distress firms have low idiosyncratic skewness when their idiosyncratic coskewness betas are negative. We will test this hypothesis by regressing distress-sorted portfolio returns on two idiosyncratic coskewness factors, $ICSK_1$ and $ICSK_2$. We will also test the robustness of our results by including other risk factors in the regressions. The regression results are reported in Table 8.

When we regress excess returns for distress-sorted portfolios on the two idiosyncratic coskewness factors, we find a striking variations in factor loadings across portfolios. The factor loadings for factor $ICSK_1$ are almost monotonically declining with default risk increasing. In contrast, the factor loadings for factor $ICSK_2$ are almost monotonically increasing with default risk. The portfolio with the highest default risk has negative loadings on factor $ICSK_1$ and positive loadings on $ICSK_2$. They are both statistically significant at 1% level. Since a positive loading on factor $ICSK_1$ and a negative loading on $ICSK_2$ will reduce expected excess return. Controlling for the two idiosyncratic coskewness factors help explain the equity return anomaly for distressed firms. The same result can be found in the regression of excess returns for a long-short portfolio holding the safest portfolio and shorting the most risky portfolio on the two idiosyncratic coskewness factors. The factor loading is positive for factor $ICSK_1$ and negative for factor $ICSK_2$. Both loadings are statistically significant. Controlling for the two idiosyncratic coskewness factors cuts alphas for the long-short portfolio roughly in half, from 1.42% to 0.64%, and it is not statistically significant.

To examine robustness of our findings, we include four standard risk factors (MKT, SMB, HML, UMD)in the regression. For the ten distress-sorted portfolios and the long-short portfolio, the factor loadings on the two idiosyncratic coskewness remain similar. Alpha for the long-short portfolio is 0.73% with a t-statistic of 1.11.

The results show that the explanatory power of the two idiosyncratic coskewness factors is large for firms on both tails of the distribution of distress measures. The adjusted R^2 in the regression of returns of the long-short portfolio based on default measures on the two idiosyncratic coskewness factors is 28%. The negative loading on $ICSK_1$ and positive loading on $ICSK_2$ help reduce the alpha for the long-short portfolio based on distress measures.

5 Conclusion

We build a theoretical model of heterogeneous skewness preference that leads to asset-pricing relationships, in addition to equity, that differ from the standard CAPM model. We show that the expected excess return on a skewed security depends on three terms. The first term is the standard risk premium in the CAPM model, the second term is the idiosyncratic coskewness betas which measures the covariance of the squared idiosyncratic shock and the market return. The last term represents the asset idiosyncratic skewness. When idiosyncratic coskewness is zero, the model is reduced to Mitton and Vorkink (2008).

We empirically show that in addition to the well known idiosyncratic skewness, the idiosyncratic coskewness measures is also an important determinant for asset returns, and provide a rational explanation on the seemingly anomalous negative relation between default risk and equity returns. Although a number of theories point toward a lower return for stocks with default risk, empirical testing of the relation between default risk and measures of idiosyncratic skewness has been slow in coming. We attempt to fill this void by estimating a model of idiosyncratic coskewness and then using idiosyncratic coskewness to explain the negative relation between default risk and equity returns.

We find a negative (negative) relation between expected equity returns and idiosyncratic coskewness if idiosyncratic coskewness betas are positive (negative). We cannot reject the hypothesis with equity data. We then construct two idiosyncratic coskewness factors to capture time series variations of equity returns sorted by idiosyncratic coskewness betas. We find that the standard market, size, value and momentum factors cannot explain the excess returns of the two idiosyncratic coskewness factors. In fact, the increase their alphas.

However, these two idiosyncratic coskewness factors can explain a large variation of distress-sorted portfolio returns. High stressed firm earn low returns because high stressed firms have high idiosyncratic coskewness betas when idiosyncratic coskewness betas are positive, and high stressed firms have low idiosyncratic coskewness betas when idiosyncratic coskewness betas are negative. Also noteworthy is that our measure of idiosyncratic coskewness cannot be explain by the idiosyncratic skewness, this suggests that both idiosyncratic skewness and idiosyncratic coskewness measures different higher moment risks.

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Table 1: Predictive Performance of Merton's Model

This table reports the number of bankruptcy and performance delistings as a function of default risk rank based on Merton's model. At the end of each month firms are assigned into portfolios according to their probability of default measures. We construct 10 portfolios containing stocks in percentiles 0-5, 5-10, 10-20, 20-40, 40-60, 60-80, 80-90, 90-95, 95-99, and 99-100 of the distribution of the default measure. Portfolio 1 contains firms with the lowest default probability measure. BD is the number of bankruptcy and liquidation delistings, and PD is the number of performance delistings. The sample period is January 1971 to December 2006.

Portfolio	Number of Bankruptcy Delistings (BD)	Number of Performance Delistings (PD)
1	0	2
2	0	3
3	1	11
4	0	23
5	1	27
6	2	42
7	3	55
8	7	67
9	20	111
10	26	102

Table 2: Summary Statistics of Returns on Distress-Sorted Portfolios

This table reports the mean, standard deviation, and skewness of monthly returns, in percentage points, on ten distress-sorted portfolios. It also reports the average size, in million dollars, of firms in each portfolio. At the end of each month firms are assigned into portfolios according to their probability of default measures. We construct 10 portfolios containing stocks in percentiles 0-5, 5-10, 10-20, 20-40, 40-60, 60-80, 80-90, 90-95, 95-99, and 99-100 of the distribution of the default measure. Portfolio 1 contains firms with the lowest default probability measure. Value-weighted realized returns in the next month are calculated for the ten portfolios. The sample period is January 1971 to December 2006.

	dp_1	dp_2	dp_3	dp_4	dp_5	dp_6	dp_7	dp_8	dp_9	dp_{10}
Mean return	0.92	0.98	1.17	1.03	1.01	0.88	0.81	0.36	0.41	−0.51
Stdev return	4.48	4.40	4.36	4.87	5.81	6.57	7.64	9.02	10.70	14.96
Skew return	−0.29	−0.48	−0.16	−0.37	−0.41	−0.27	0.15	0.49	0.86	0.81
Mean size	4.90	4.33	2.80	1.43	0.71	0.35	0.18	0.11	0.10	0.07

Table 3: Equity Returns on Distress-Sorted Portfolios

We sort all stocks on the predicted default probability from Merton's Model and divide them into 10 portfolios on percentile cutoffs. The ten portfolios containing stocks in percentiles 0–5, 5–10, 10–20, 20–40, 40–60, 60–80, 80–90, 90–95, 95–99, and 99-100 of the distribution of the default measure. The portfolio contains the 0 to 5th percentile is denoted by 0005, and the portfolio contains the 99th to 100th percentile is denoted by 9900. The hedge portfolio that longs the 0005 and shorts 9900 is denoted by $LS0599$. This table reports the regression results from regressions of value-weighted excess returns on a constant, market excess return (*MKT*), three (*MKT, SMB, HML*) Fama-French factors, and four (*MKT, SMB, HML, UMD*) factors. The sample period is January 1971 to December 2006. Panel A shows alphas (in monthly percent units) from these regressions. Panel B, C, and D report factor loadings from the CAMP, three-factor and four-factor regressions respectively. The robust Newey-West t-statistics are reported in parentheses.

Portfolios	0005	0510	1020	2040	4060	6080	8090	9095	9599	9900	$LS0599$
Panel A. Portfolio Alphas											
Mean excess return	0.43	0.50	0.69	0.55	0.53	0.40	0.33	−0.12	−0.08	−0.99	1.42
	(1.79)	(2.20)	(3.48)	(2.55)	(2.13)	(1.30)	(0.94)	(0.28)	(0.14)	(1.43)	(2.19)
CAPM alpha	−0.01	0.04	0.22	0.02	−0.07	−0.25	−0.36	−0.86	−0.89	−1.95	1.94
	(0.08)	(0.40)	(3.14)	(0.25)	(0.56)	(1.50)	(1.54)	(2.95)	(2.18)	(3.41)	(3.16)
3-factor alpha	0.20	0.16	0.24	−0.08	−0.33	−0.55	−0.79	−1.30	−1.41	−2.56	2.76
	(2.18)	(1.78)	(3.98)	(0.83)	(2.95)	(3.52)	(3.80)	(4.67)	(3.93)	(4.67)	(4.90)
4-factor alpha	0.01	−0.01	0.20	−0.01	−0.08	−0.09	−0.19	−0.54	−0.44	−1.37	1.38
	(0.11)	(0.09)	(2.82)	(0.14)	(0.76)	(0.84)	(0.99)	(2.25)	(1.43)	(2.28)	(2.27)
Panel B. CAMP factor loading											
MKT	0.87	0.89	0.92	1.03	1.18	1.27	1.35	1.46	1.60	1.88	−1.01
	(21.07)	(27.72)	(36.35)	(39.63)	(25.88)	(21.09)	(18.50)	(15.32)	(12.69)	(10.34)	(4.81)
Panel C. Three-factor loadings											
MKT	0.83	0.87	0.95	1.07	1.24	1.33	1.42	1.46	1.62	1.81	−1.01
	(22.74)	(25.67)	(43.42)	(53.99)	(42.76)	(22.70)	(21.09)	(14.11)	(13.10)	(9.06)	(4.36)
SMB	−0.25	−0.15	−0.13	0.04	0.30	0.40	0.60	0.90	1.01	1.55	−1.81
	(6.81)	(3.46)	(5.11)	(0.96)	(4.14)	(2.85)	(3.21)	(3.88)	(3.91)	(6.20)	(6.57)
HML	−0.30	−0.16	−0.01	0.15	0.38	0.44	0.61	0.59	0.71	0.78	−1.08
	(5.10)	(3.72)	(0.23)	(2.73)	(4.92)	(3.37)	(3.78)	(2.74)	(2.35)	(4.01)	(4.72)
Panel D. Four-factor loadings											
MKT	0.85	0.89	0.95	1.06	1.20	1.26	1.34	1.36	1.48	1.65	−0.79
	(29.04)	(31.64)	(44.59)	(50.64)	(40.07)	(29.63)	(27.92)	(18.05)	(17.85)	(12.57)	(5.46)
SMB	−0.26	−0.16	−0.13	0.04	0.03	0.40	0.61	0.92	1.03	1.56	−1.83
	(8.49)	(5.02)	(4.92)	(1.17)	(6.91)	(5.10)	(5.17)	(6.40)	(6.43)	(8.20)	(8.91)
HML	−0.26	−0.13	0.00	0.14	0.32	0.34	0.48	0.41	0.49	0.51	−0.77
	(4.96)	(3.03)	(0.04)	(2.85)	(6.06)	(4.95)	(5.16)	(3.25)	(2.42)	(3.21)	(4.63)
UMD	0.18	0.16	0.04	−0.06	−0.25	−0.45	−0.57	−0.73	−0.94	−1.16	1.34
	(3.35)	(5.56)	(1.21)	(1.84)	(5.06)	(9.09)	(6.31)	(6.32)	(5.61)	(5.67)	(5.71)

Table 4: Equity Returns on Portfolios with Positive Idiosyncratic Coskewness Betas

We sort stocks based on estimated idiosyncratic coskewness betas which have positive values, and divide them into 10 decile portfolios. 0010 denotes the portfolio with the lowest idiosyncratic coskewness beta, i.e. the 0–10 percentile, and 9900 denotes the portfolio with the highest idiosyncratic coskewness beta, i.e. the 90–100 percentile. The hedge portfolio that longs 0010 and shorts 9000 is denoted by $LS1090$. This table reports results from regressions of value-weighted excess returns on a constant, market excess return (MKT), three (MKT, SMB, HML) Fama-French factors, and four (MKT, SMB, HML, UMD) factors. The sample period is January 1971 to December 2006. Panel A shows alphas (in monthly percent units) from these regressions. Panel B reports factor loadings from the four-factor regressions. The robust Newey-West t-statistics are reported in parentheses.

Portfolios	0010	1020	2030	3040	4050	5060	6070	7080	8090	9000	*LS*1090
Panel A. Portfolio Alphas											
Mean excess return	0.66	0.45	0.51	0.56	0.64	0.52	0.64	0.58	0.40	−0.15	0.81
	(3.20)	(2.12)	(2.54)	(2.32)	(2.56)	(1.79)	(2.13)	(1.54)	(0.85)	(0.29)	(1.87)
CAPM alpha	0.24	0.01	0.04	0.02	0.04	−0.13	−0.04	−0.17	−0.43	−0.98	1.21
	(2.39)	(0.12)	(0.58)	(0.31)	(0.41)	(1.00)	(0.26)	(0.81)	(1.38)	(2.95)	(3.14)
3-factor alpha	0.17	−0.03	0.02	−0.01	0.03	−0.11	−0.05	−0.05	−0.25	−0.95	1.12
	(2.28)	(0.39)	(0.31)	(0.15)	(0.28)	(0.91)	(0.39)	(0.29)	(1.00)	(3.75)	(4.29)
4-factor alpha	0.21	−0.01	0.01	0.02	0.05	−0.03	0.04	0.04	−0.10	−0.77	0.98
	(2.54)	(0.12)	(0.20)	(0.18)	(0.56)	(0.27)	(0.27)	(0.23)	(0.44)	(3.49)	(4.02)
Panel B. Four-factor loadings											
MKT	0.91	0.93	0.96	1.07	1.15	1.18	1.21	1.24	1.29	1.28	−0.37
	(58.44)	(37.32)	(37.92)	(37.44)	(37.75)	(28.07)	(31.10)	(16.74)	(18.79)	(15.42)	(3.96)
SMB	−0.27	−0.18	−0.08	−0.03	0.16	0.30	0.54	0.68	0.95	1.29	−1.57
	(9.93)	(6.08)	(3.20)	(0.48)	(4.02)	(7.07)	(8.24)	(8.34)	(11.36)	(14.46)	(15.09)
HML	0.13	0.09	0.04	0.06	−0.01	−0.10	−0.08	−0.31	−0.44	−0.26	0.40
	(2.74)	(2.39)	(1.39)	(1.07)	(0.22)	(2.06)	(1.21)	(2.79)	(2.48)	(1.29)	(1.63)
UMD	−0.04	0.02	0.01	−0.03	−0.03	−0.08	−0.09	−0.08	−0.15	−0.18	0.14
	(1.53)	(0.69)	(0.21)	(0.70)	(0.81)	(1.64)	(2.24)	(1.11)	(1.41)	(1.58)	(1.08)

Table 5: Equity Returns on Portfolios with Negative Idiosyncratic Coskewness Betas

We sort stocks based on estimated idiosyncratic coskewness betas which have negative values, and divide them into 10 decile portfolios. 0010 denotes the portfolio with the lowest idiosyncratic coskewness beta, i.e. the 0–10 percentile, and 9900 denotes the portfolio with the highest idiosyncratic coskewness beta, i.e. the 90–100 percentile. The hedge portfolio that longs 0010 and shorts 9000 is denoted by $LS1090$. This table reports results from regressions of value-weighted excess returns on a constant, market excess return (MKT), three (MKT, SMB, HML) Fama-French factors, and four (MKT, SMB, HML, UMD) factors. The sample period is January 1971 to December 2006. Panel A shows alphas (in monthly percent units) from these regressions. Panel B reports factor loadings from the four-factor regressions. The robust Newey-West t-statistics are reported in parentheses.

Portfolios	0010	1020	2030	3040	4050	5060	6070	7080	8090	9000	$LS1090$
Panel A. Portfolio Alphas											
Mean excess return	−0.09	0.32	0.11	0.49	0.31	0.60	0.53	0.62	0.38	0.54	−0.63
	(0.25)	(0.92)	(0.39)	(1.70)	(1.10)	(2.61)	(2.03)	(2.80)	(1.68)	(2.42)	(2.00)
CAPM alpha	−0.76	−0.32	−0.53	−0.12	−0.30	0.06	0.02	0.14	−0.06	0.12	−0.88
	(2.72)	(1.15)	(3.25)	(0.78)	(2.19)	(0.44)	(0.21)	(1.17)	(0.59)	(1.04)	(2.74)
3-factor alpha	−0.81	−0.35	−0.52	−0.13	−0.36	−0.01	−0.02	0.08	−0.14	0.04	−0.85
	(3.92)	(1.46)	(3.66)	(0.87)	(2.58)	(0.05)	(0.13)	(0.75)	(1.48)	(0.37)	(3.76)
4-factor alpha	−0.56	−0.15	−0.41	−0.06	−0.28	−0.03	0.13	0.09	−0.12	0.05	−0.61
	(2.53)	(0.54)	(2.88)	(0.38)	(1.98)	(0.25)	(1.17)	(0.78)	(1.29)	(0.41)	(2.48)
Panel B. Four-factor loadings											
MKT	1.06	1.07	1.11	1.10	1.15	1.08	1.02	0.98	0.95	0.89	0.17
	(15.91)	(24.70)	(22.34)	(30.31)	(22.82)	(27.42)	(32.68)	(26.15)	(48.18)	(31.38)	(2.05)
SMB	0.99	0.73	0.57	0.40	0.29	0.02	−0.08	−0.09	−0.21	−0.17	1.16
	(10.54)	(9.64)	(7.46)	(8.03)	(4.45)	(0.24)	(1.30)	(1.65)	(4.59)	(5.35)	(11.57)
HML	−0.11	−0.09 −	0.12	−0.06	0.04	0.12	0.04	0.10	0.16	0.15	−0.26
	(0.99)	(1.24)	(1.75)	(1.08)	(0.54)	(1.22)	(1.00)	(1.75)	(2.80)	(2.81)	(1.84)
UMD	−0.24	−0.19	−0.10	−0.07	−0.08	0.03	−0.15	−0.01	−0.02	−0.06	−0.23
	(2.95)	(2.22)	(2.60)	(1.89)	(2.51)	(0.53)	(3.57)	(0.30)	(0.50)	(0.16)	(2.23)

Table 6: Equity Returns for Portfolios Sorted by Idiosyncratic Skewness

This table reports results of intercepts from regressions of value-weighted excess returns for idiosyncratic skewness-sorted portfolios on a constant, market excess return (*MKT*), three (*MKT, SMB, HML*) Fama-French factors, and four (*MKT, SMB, HML, UMD*) factors. The sample period is January 1971 to December 2006.

Portfolios	0010	1020	2030	3040	4050	5060	6070	7080	8090	9000
Mean excess return	0.36	0.62	0.61	0.61	0.53	0.61	0.53	0.61	0.66	0.71
	(1.79)	(2.76)	(2.78)	(2.70)	(2.25)	(2.73)	(2.38)	(2.39)	(2.33)	(2.83)
CAPM alpha	−0.12	0.14	0.11	0.09	0.01	0.08	−0.02	0.07	0.07	0.22
	(1.92)	(2.07)	(2.13)	(1.39)	(0.13)	(1.37)	(0.28)	(0.66)	(1.52)	(1.50)
3-factor alpha	−0.11	0.13	0.12	0.09	0.00	0.07	−0.11	−0.01	−0.00	0.12
	(1.53)	(1.95)	(2.50)	(1.40)	(0.02)	(1.12)	(1.90)	(0.07)	(0.02)	(1.00)
4-factor alpha	0.05	0.18	0.14	0.08	0.02	−0.06	−0.13	−0.02	−0.10	0.12
	(0.89)	(2.19)	(2.66)	(1.25)	(0.26)	(1.05)	(1.75)	(0.24)	(0.86)	(1.03)

Table 7: Test Intercepts from the Fama-French Model and the Carhart Model

This table reports the results from multivariate tests on intercepts from time-series regressions with the three FamaFrench factors and the Carhart four factors including the momentum factor. We also include the two idiosyncratic coskewness factors in the regressions. The test-statistic is the Gibbons–Ross–Shanken F-test statistic distributed as $F \sim (N, T-N-K)$, where T is the number of observations, N is the number of portfolios and K is the number of factors. The p-values of the test statistics are presented in parentheses. The sample period is January 1971 to December 2006.

Portfolios	Number of Portfolios	F-test for Fama-French Three Factors	F-test for Fama-French Three Factors and Two ICSK Factors	F-test for Carhart Four Factors	F-test for Carhart Four Factors and Two ICSK Factors
industrial	30	15.45	11.80	12.68	10.00
		(0.00)	(0.00)	(0.00)	(0.00)
industrial	48	20.86	13.19	14.91	10.69
		(0.00)	(0.00)	(0.00)	(0.00)
Size	10	5.80	2.64	5.19	2.61
		(0.00)	(0.00)	(0.00)	(0.00)
Book/Market	10	4.96	1.95	3.39	1.31
		(0.00)	(0.04)	(0.00)	(0.22)
Momentum	10	14.90	7.88	8.85	5.52
		(0.00)	(0.04)	(0.00)	(0.22)
Positive idiosyncratic	10	8.99	1.88	6.39	1.83
coskewness beta		(0.00)	(0.06)	(0.00)	(0.05)
Negative idiosyncratic	10	8.37	2.50	4.06	1.51
coskewness beta		(0.00)	(0.01)	(0.00)	(0.13)

Table 8: Explaining Equity Returns on Distress-Sorted Portfolios

This table reports the regression results from regressions of value-weighted excess returns of distress-sorted portfolios on two idiosyncratic coskewness factors ($ICSK_1$, $ICSK_2$), and six factors (*MKT, SMB, HML, UMD*, $ICSK_1$, $ICSK_2$). The sample period is January 1971 to December 2006. Panel A shows alphas (in monthly percent units) from these regressions. Panel B and C report factor loadings and R^2-adjusted from the two-factor and six-factor regressions respectively. The robust Newey-West t-statistics are reported in parentheses.

Portfolios	0005	0510	1020	2040	4060	6080	8090	9095	9599	9900	*LS*0599
Panel A. Portfolio Alphas											
two-factor alpha	−0.18	−0.08	0.13	0.04	0.10	0.05	0.04	−0.28	−0.15	−0.82	0.64
	(1.57)	(0.86)	(1.88)	(0.47)	(0.76)	(0.25)	(0.17)	(0.85)	(0.34)	(1.38)	(1.01)
six-factor alpha	−0.12	−0.07	0.16	−0.02	−0.09	−0.01	−0.08	−0.34	−0.13	−0.85	0.73
	(1.27)	(0.88)	(2.25)	(0.28)	(0.93)	(0.11)	(0.41)	(1.29)	(0.39)	(1.30)	(1.11)
Panel B. Two-factor loadings											
$ICSK_1$	0.12	0.05	0.06	0.01	−0.06	−0.11	−0.15	−0.30	−0.35	−0.56	0.68
	(3.07)	(1.74)	(5.03)	(0.31)	(0.83)	(1.26)	(1.08)	(1.95)	(1.81)	(3.08)	(3.14)
$ICSK_2$	−0.02	−0.05	−0.02	0.02	0.07	0.11	0.17	0.18	0.25	0.36	−0.38
	(0.68)	(2.16)	(1.62)	(1.61)	(2.53)	(3.65)	(3.72)	(3.16)	(3.19)	(3.30)	(2.95)
R^2−adjusted	0.20	0.12	0.15	0.01	0.10	0.18	0.18	0.25	0.24	0.24	0.28
Panel C. Six-factor loadings											
MKT	−0.10	−0.08	−0.03	0.07	0.21	0.24	0.30	0.28	0.38	0.47	−0.56
	(3.93)	(2.94)	(1.80)	(3.00)	(6.34)	(5.36)	(5.58)	(3.74)	(4.25)	(4.57)	(5.12)
SMB	−0.04	−0.05	−0.06	0.06	0.33	0.27	0.42	0.58	0.52	0.71	−0.76
	(0.96)	(1.02)	(1.53)	(1.37)	(6.24)	(3.84)	(3.65)	(4.31)	(3.07)	(2.10)	(2.18)
HML	−0.31	−0.05	−0.02	0.13	0.31	0.37	0.52	0.50	0.61	0.72	−1.03
	(8.19)	(3.49)	(0.70)	(3.15)	(6.81)	(5.37)	(5.92)	(4.17)	(3.66)	(4.26)	(6.08)
UMD	0.16	0.15	0.03	−0.06	−0.25	−0.43	−0.55	−0.70	−0.88	−1.06	1.22
	(3.78)	(5.41)	(1.01)	(1.79)	(4.92)	(8.40)	(5.74)	(6.05)	(5.55)	(5.82)	(6.09)
$ICSK_1$	0.13	0.05	0.04	0.02	0.02	−0.05	−0.07	−0.17	−0.24	−0.41	0.54
	(8.00)	(2.15)	(2.65)	(0.91)	(0.69)	(1.91)	(1.12)	(2.83)	(2.47)	(3.02)	(3.81)
$ICSK_2$	−0.01	−0.03	−0.01	0.01	0.01	0.04	0.08	0.06	0.12	0.19	−0.20
	(0.35)	(1.53)	(0.51)	(0.80)	(0.35)	(1.71)	(2.63)	(1.23)	(1.98)	(2.04)	(1.97)
R^2−adjusted	0.45	0.30	0.17	0.11	0.51	0.60	0.55	0.53	0.51	0.40	0.48

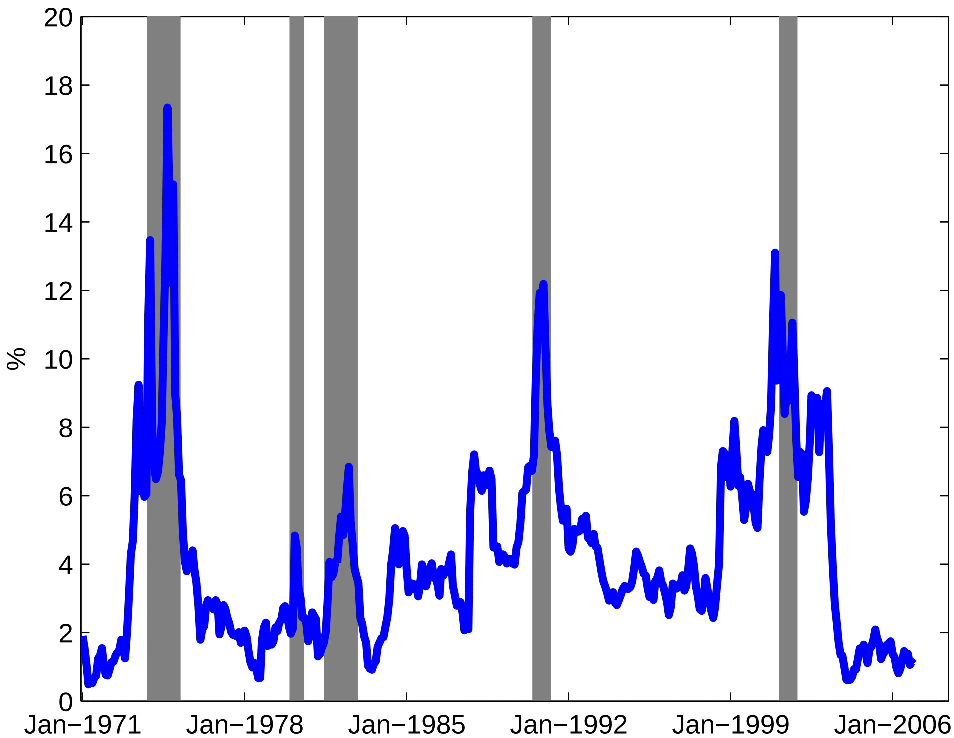

Figure 1: Average Default Probability

This graph plot monthly average default probabilities. The shaded areas denote recession periods, as defined by NBER. The sample period is January 1971 to December 2006.

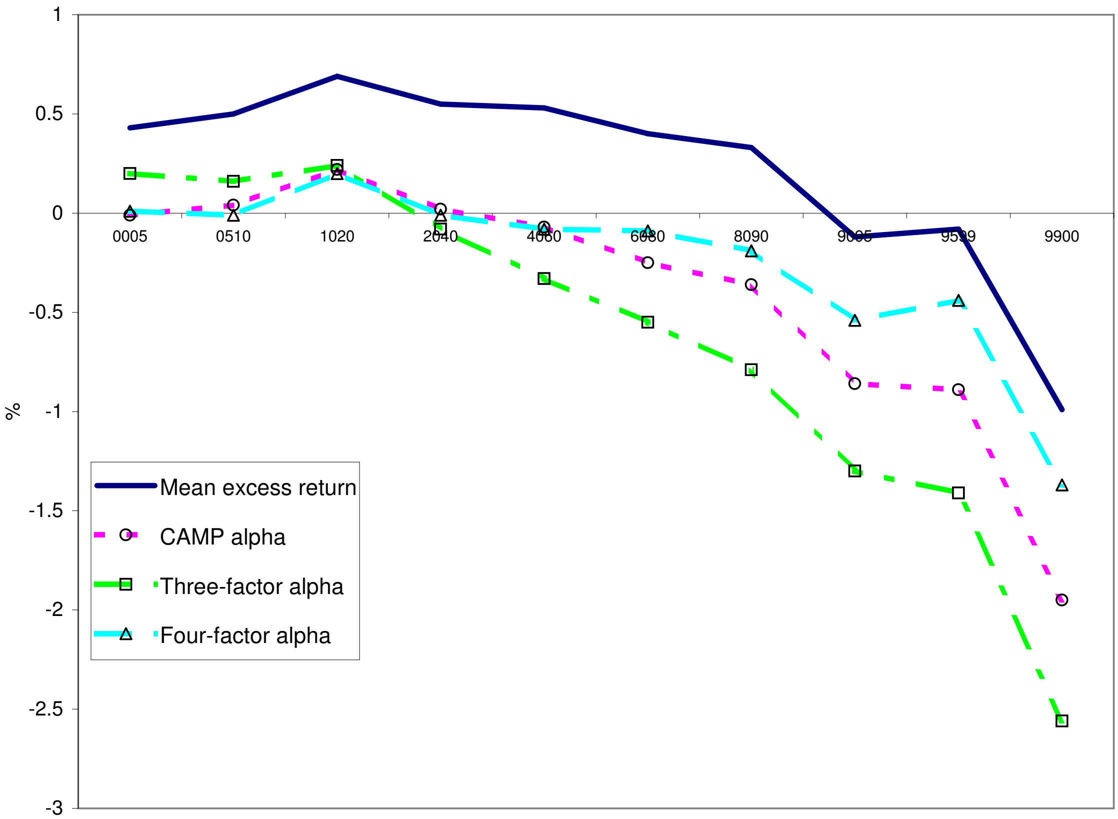

Figure 2: Alphas of Distress-Sorted Portfolios

This graph plot monthly excess returns of 10 distress-sorted portfolios, and alphas with respect to the CAMP, the three-factor model of Fama-French (1993), and four-factor model of Carhart (1997). The sample period is January 1971 to December 2006. Portfolios are formed at the beginning of each month during the sample period.

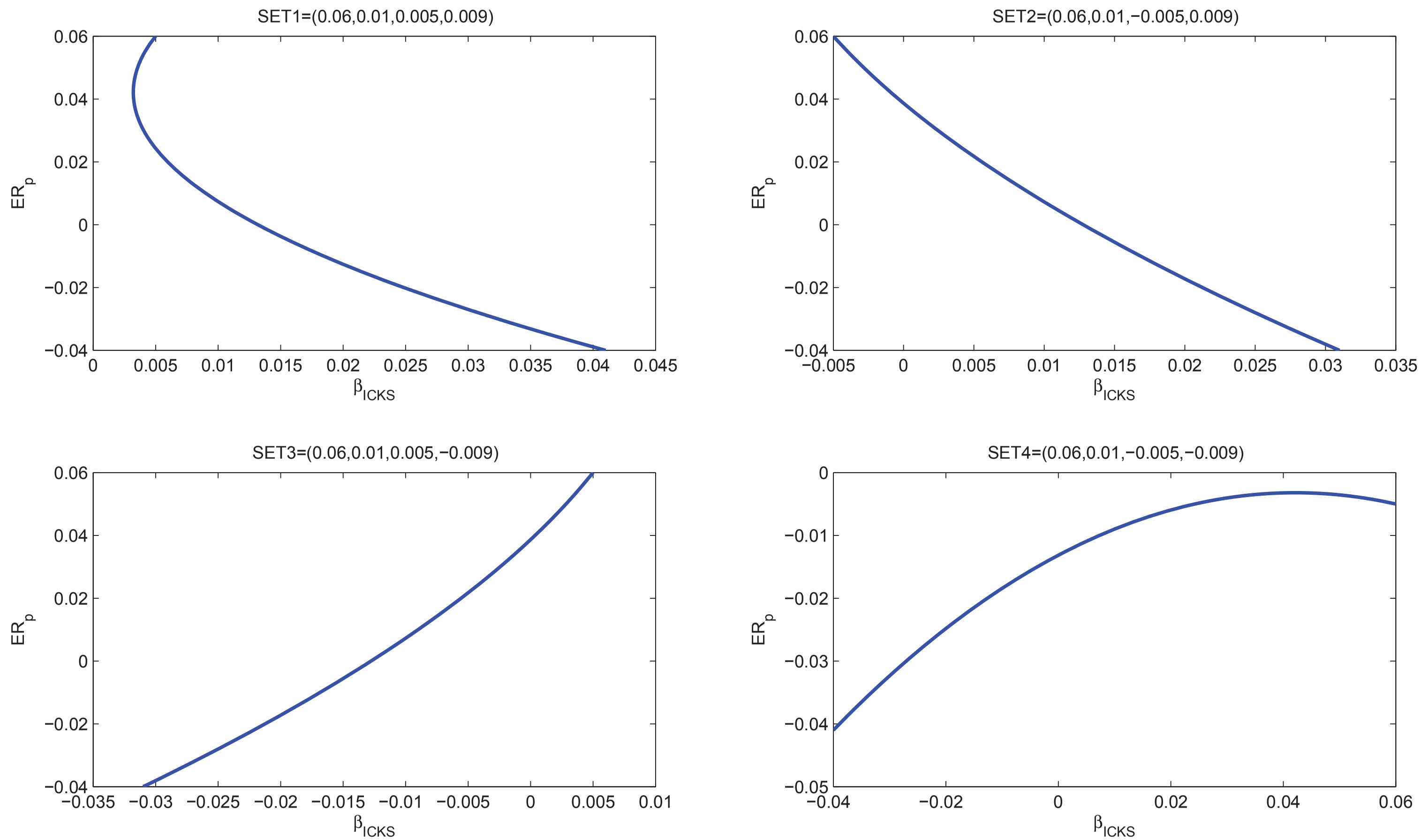

Figure 3: Relation between Expected Returns and Idiosyncratic Coskewness Betas 1

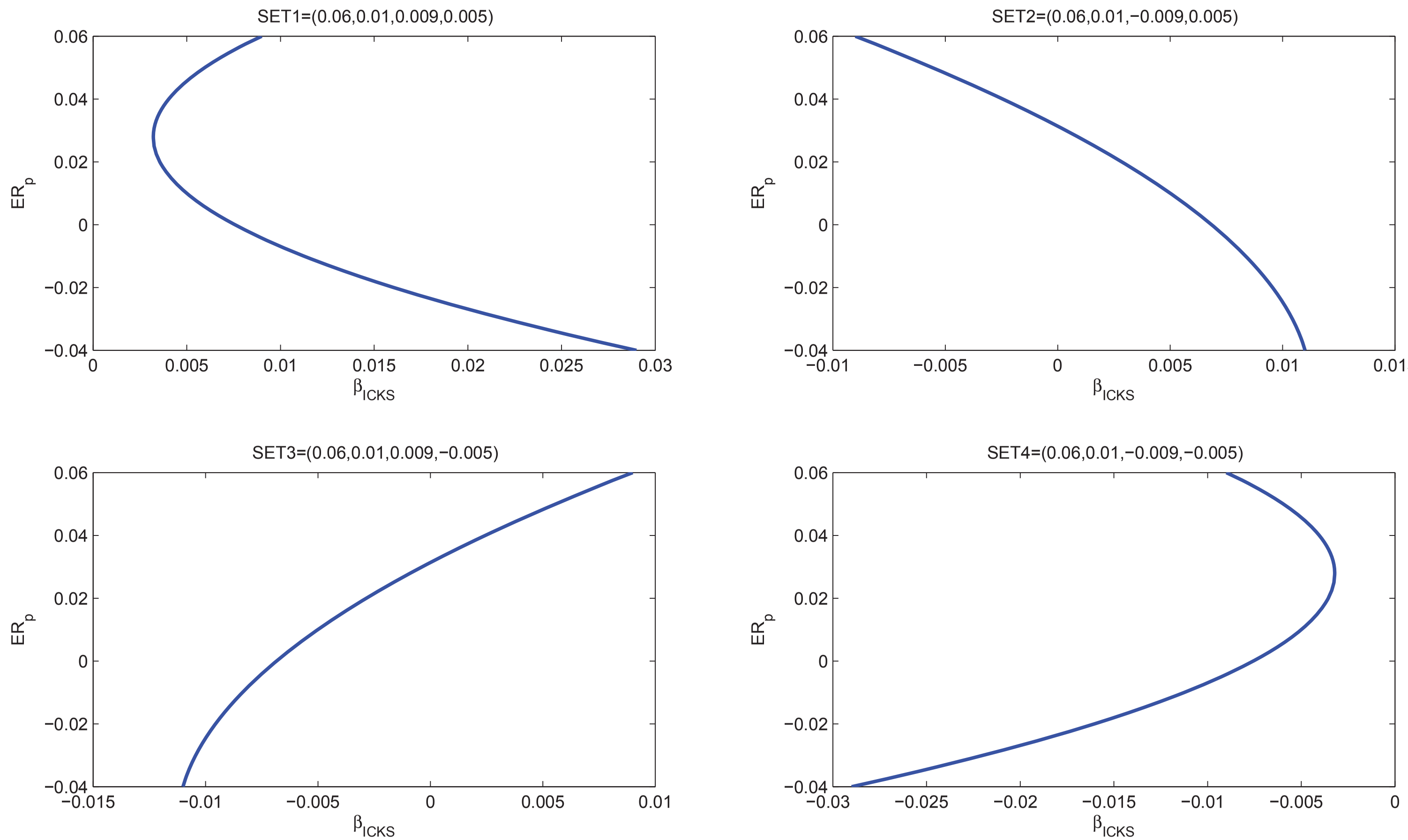

Figure 4: Relation between Expected Returns and Idiosyncratic Coskewness Betas 2

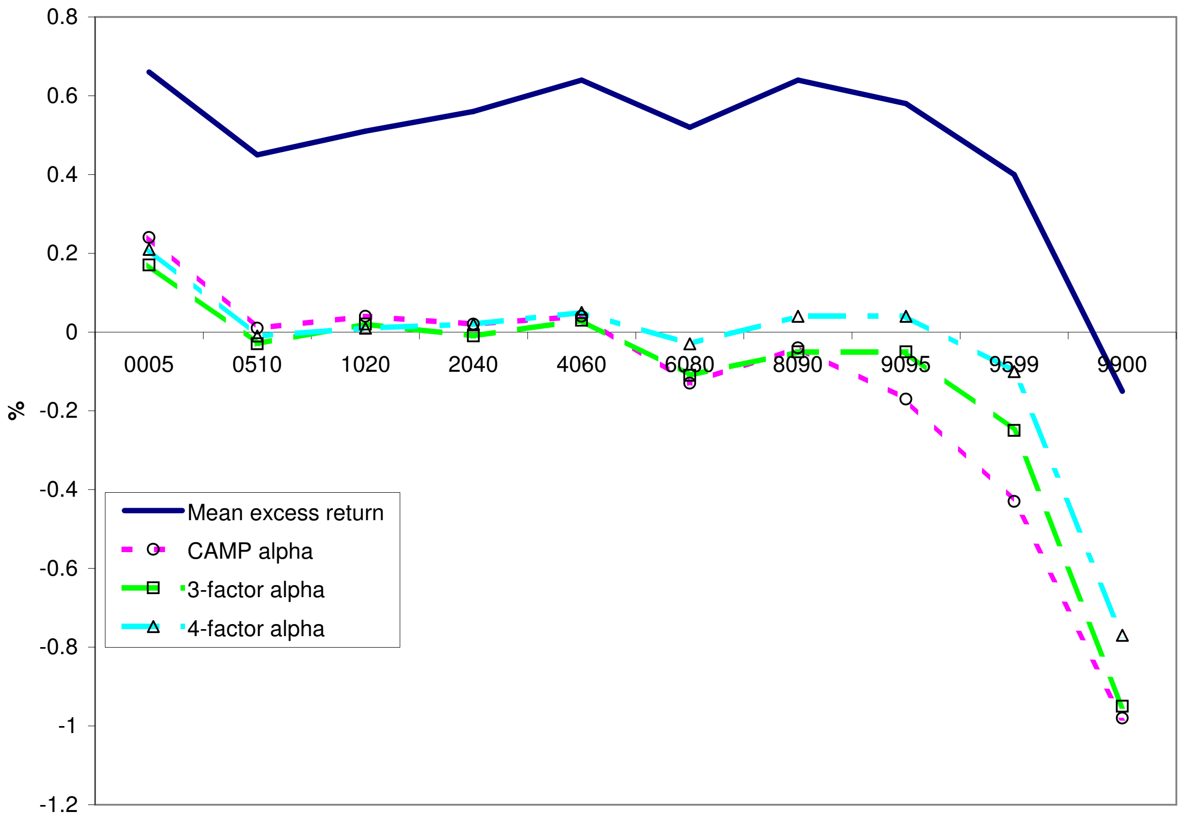

Figure 5: Alphas of Portfolios Sorted by Idiosyncratic Coskewness Betas (Positive Values)

This graph plot monthly excess returns of 10 portfolios sorted by idiosyncratic coskewness betas (positive values), and alphas with respect to the CAMP, the three-factor model of Fama-French (1993), and four-factor model of Carhart (1997). The sample period is January 1971 to December 2006. Portfolios are formed at the beginning of each month during the sample period.

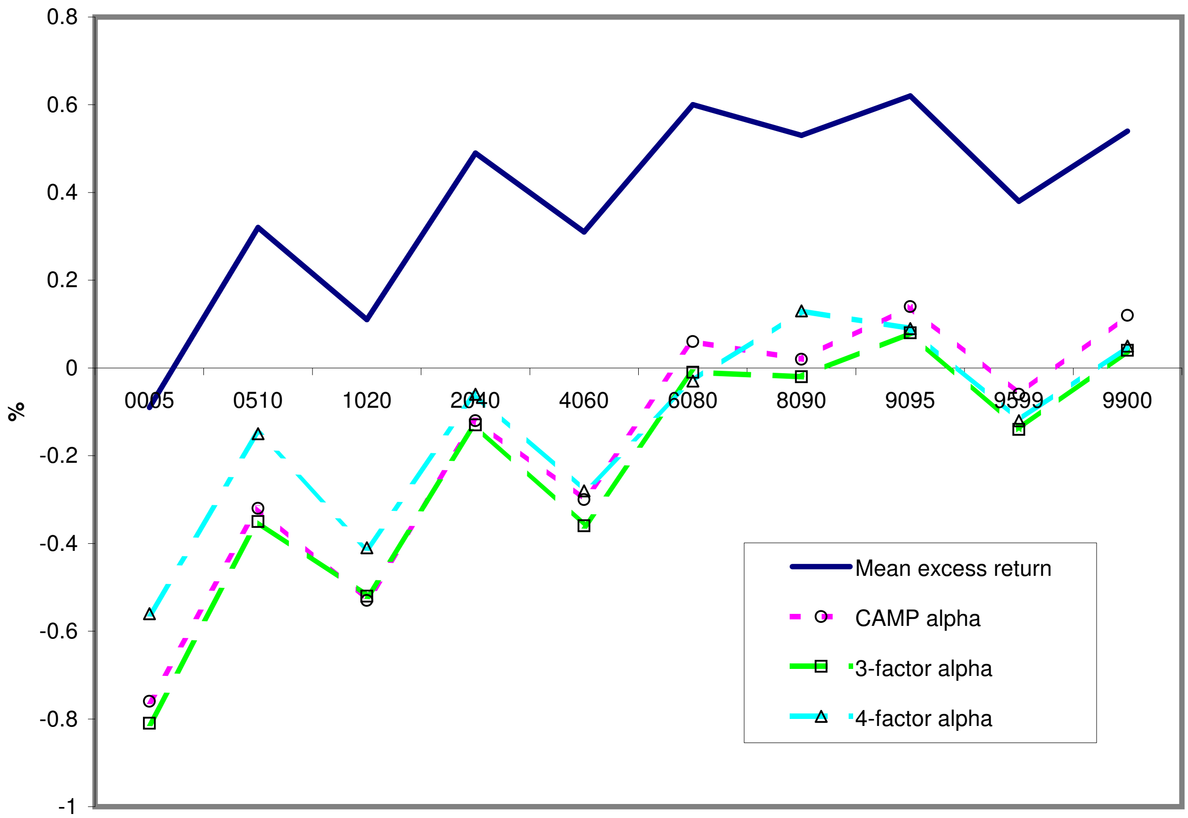

Figure 6: Alphas of Portfolios Sorted by Idiosyncratic Coskewness Betas (Negative Values)

This graph plot monthly excess returns of 10 portfolios sorted by idiosyncratic coskewness betas (negative values), and alphas with respect to the CAMP, the three-factor model of Fama-French (1993), and four-factor model of Carhart (1997). The sample period is January 1971 to December 2006. Portfolios are formed at the beginning of each month during the sample period.