Investor Overconfidence and the Forward Discount Puzzle *

Bing Han†

David Hirshleifer‡

Tracy Yue Wang§

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†Fisher College of Business, The Ohio State University, 2100 Neil Avenue, Columbus, OH 43210-1144; han_184@cob.osu.edu, 614-292-1875;

‡Fisher College of Business, The Ohio State University, 2100 Neil Avenue, Columbus, OH 43210-1144; hirshleifer_2@cob.osu.edu, 614-292-5174;

§Carlson School of Management, University of Minnesota, 321 19th Avenue South, Minneapolis, MN 55455; tracywang@csom.umn.edu, 612-624-5869.

ABSTRACT

This paper offers an explanation for the forward discount puzzle in foreign exchange markets based upon investor overconfidence. In contrast with behavioral biases conjectured specifically to explain the puzzle, overconfidence is a well-documented psychological phenomenon that has been found to be consistent with several trading and return patterns in securities markets. In our model, overconfident individuals overreact to their information about future inflation. The spot rate and forward discount differentially reflect such overreaction; as a result, the forward discount forecasts reversal in the spot rate. The model predicts how the forward discount bias varies with time horizon, time-series versus cross-sectional test method, and shifts in volume and volatility.

1 Introduction

Nominal interest rates should reflect investor expectations about future inflation. If investors rationally foresee future inflation, then currencies in which bonds offer high nominal interest rates should on average depreciate relative to low-nominal-interest-rate currencies. Furthermore, when the interest rate differential is higher than usual, the rate of depreciation should be higher than usual. A strong empirical finding, however, is that at times when short-term nominal interest rates are high in one currency relative to another, that currency subsequently *appreciates* on average (see, e.g., surveys of Hodrick, 1988; Lewis, 1995; Engel, 1996). An equivalent finding is that the forward discount (defined as the difference between the forward and spot exchange rates) *negatively* forecasts subsequent exchange rate changes, a pattern known as the forward discount puzzle.

The most extensively explored explanation for the forward discount puzzle is that it reflects time-varying rational premia for systematic risk (e.g., Fama, 1984). However, the survey of Hodrick (1987) concludes that "we do not yet have a model of expected returns that fits the data" in foreign exchange markets. The more recent survey of Engel (1996) similarly concludes that rational risk pricing has not explained the forward discount bias, and suggests that an approach based upon imperfect rationality can potentially offer new insights about the puzzle.

In this paper we propose an explanation for the forward discount puzzle based upon investor overconfidence, a well-documented psychological bias. A growing analytical and empirical literature has argued that investor overconfidence can explain puzzling patterns of return predictability, return volatility, volume of trading, and individual trading profits in securities markets (Hirshleifer (2001) reviews some recent models of investor overconfidence). If a systematic bias such as overconfidence causes anomalies in stock markets, it should also leave footprints in bond and foreign exchange markets. Thus, a behavioral explanation for anomalies is more credible if it can explain a range of patterns across different kinds of markets, thereby obviating the need to tailor a different theory for each anomaly and type of market.

In our model, overconfident individuals think that the precision of their information signal about the future inflation differential is greater than it actually is. As a result, investor expectations overreact to the signal. This causes both the forward and spot exchange rates to overshoot in the same direction. However, the consumption price level and the spot exchange rate are influenced by a transactions demand for money, whereas forward rates are more heavily influenced by speculative considerations, i.e., the expected return from holding domestic or foreign bonds. Therefore, the forward rate overshoots more than the spot rate, which implies that the forward discount serves as a measure of investor overreaction (and is, in a sense we will make precise, a better measure of overreaction than the forward or spot rates alone). Later, the overreaction in the spot rate is on average reversed. The forward discount is a predictor of this correction, and hence on average can be negatively related to the subsequent exchange rate depreciations.

The sign of the slope coefficient in a regression of the future spot rate change on the forward discount depends on two opposing effects. The overreaction effect described above favors a negative coefficient. On the other hand, if the information investors receive is authentic, and there is no overreaction in the spot rate, then a higher forward discount positively predicts future spot rate changes — this is the conventional effect which makes the empirical findings a puzzle. We show that over short horizons, the overreaction-correction effect dominates, but over long horizons, the positive conventional effect eventually dominates. Intuitively, over time mispricing in the spot exchange rate attenuates, whereas the effects of fully foreseeable differences in expected money growth rates across different currencies accumulate. This model implication is consistent with evidence that the forward discount regression coefficients switch from negative to positive at long horizons (e.g., Gourinchas and Tornell, 2004; Meredith and Chinn, 2004).

Similarly, there is a tendency for countries with high average interest rate differentials relative to the U.S. over long periods of time also to have high average depreciation relative to the dollar (e.g., Cochrane, 1999). The contrast between this cross-sectional result and the forward discount puzzle in the time-series regressions is consistent with our model. A substantial part of the long-run inflation differential across countries is foreseeable, fairly constant, and not a matter of subjective judgment — what could be called the known component of the inflation differential. The long-run mean interest rate differentials between countries tend to reflect heavily the known component, and to average out the transitory effects of mispricing. In contrast, a time-series regression of exchange rate depreciation on the forward discount tends to eliminate (throw into the constant term) the known, predictable components of the inflation differentials and focus on judgment-sensitive fluctuations in expectations. Thus, the forward discount bias should be less pronounced in a cross-sectional regression of short-term future depreciation on the short-term forward discount than in time-series tests.

Some economists have suggested that exchange rate overshooting of underlying economic fundamentals can take the form of deviations from Purchasing Power Parity (PPP). For example, in the model of Dornbusch (1976), the spot exchange rate overshoots because of differing speeds of price adjustment in goods and asset markets. Our approach allows for, but does not require, violations of PPP. In our model overreaction in investor expectations affects both country price levels and spot exchange rates. In consequence, there is overshooting in both exchange rates and country price levels even if the exchange rate and country price levels are perfectly aligned (so that PPP holds). Furthermore, in our model deviations from PPP alone cannot generate forward discount bias for reasonable parameters. However, we show that when investors are overconfident, PPP violations amplify the overreaction-correction effect associated with overconfidence, leading to a more severe forward discount bias.

Our application of overconfidence to foreign exchange markets is motivated by several stylized facts. First, a large body of evidence from cognitive psychological experiments and surveys indicates that people are overconfident in various settings.[1] Second, stock markets provide evidence consistent with investor overconfidence.[2] Third, there is evidence that investors in foreign exchange markets are influenced by the overconfidence bias. The high trading volume and exchange rate volatility in foreign exchange markets are consistent with overconfidence. Oberlechner and Osler (2004) provide direct survey evidence that currency market professionals tend to overestimate the precision of their information signals. There is also evidence that currency traders' expectations about future depreciations tend to overreact, and that such overreaction is positively related to the forward discount (e.g., Froot and Frankel, 1989). We will show that this evidence is consistent with overconfident investors overreacting to their information and the overreaction being reflected in the forward discount.

Motivated by the lack of empirical support for a rational risk explanation for the forward discount puzzle, a few papers have offered models featuring investor irrationality. For example, Frankel and Froot (1990a) consider a setting in which two groups of foreign exchange forecasters deviate from rationality in different ways. Their model generates bubble-like dynamics for spot exchange rates, but does not specifically address the forward discount puzzle. Mark and Wu (1998) study the noise trader model of DeLong et al (1990) in the pricing of foreign currencies. Noise trader beliefs about first moments of returns are exogenously distorted. Gourinchas and Tornell (2004) find that the forward discount puzzle can result from investor underreactions to interest rate innovations.

In evaluating this line of research, some observers (e.g., McCallum 1994) emphasize

[1]According to DeBondt and Thaler (1995), overconfidence is "perhaps the most robust finding in the psychology of judgement." Individuals from various professional fields have been observed to overestimate the accuracy of their judgments and their abilities to perform judgment tasks, especially difficult ones in which the feedback on their decisions is deferred or inconclusive. Odean (1998) and Hirshleifer (2001) review psychological evidence regarding overconfidence.

[2]Individual investors trade actively and on average lose money on their trades, consistent with overconfidence (e.g., DeBondt and Thaler, 1985; Barber and Odean, 2000). Investor overconfidence has been proposed as an explanation for several patterns in stock markets, such as momentum, long-term reversals (e.g., Daniel, Hirshleifer and Subrahmanyam, 1998; 2001), aggressive trading, and high return volatility (e.g., Odean, 1998).

the need for behavioral approaches to provide an underlying motivation for their assumptions about the form of irrationality or noise trading. Our paper differs from past research in basing its assumptions about belief formation on evidence from psychology, and in considering whether the assumed psychological bias has realistic implications for other security markets. This paper thereby contributes to the imperfect rationality approach to explaining the forward discount puzzle in the following ways.

First, we show that the average negative relationship between the forward discount and future exchange rate changes is a natural consequence of a well-documented cognitive bias, overconfidence. We derive price relationships from investor beliefs, rather than directly making assumptions about trading behavior. Furthermore, we do not assume that belief errors have a particular correlation with the forward discount, but rather derive this correlation from the psychological premise.

Second, our psychology-based explanation of the forward discount puzzle is not developed ex post specifically for the purpose of solving this puzzle. Investor overconfidence has been used to explain a range of other cross-sectional and time-series patterns of return predictability in securities markets as well as patterns in volume, volatility, and investor trading profits.

Third, our approach explains why the forward discount tends to negatively predict spot exchange rate changes over short horizons, and less negatively or positively at longer horizons.

Finally, our model provides several new empirical predictions about forward discount regressions. For example, our model predicts that the magnitude of the forward discount bias will change over time as the level of investor overconfidence shifts. The forward discount regression coefficient will be more negative in periods that are more subject to investor overconfidence (e.g., periods of high trading volume, or exchange rate volatility, or foreign exchange forecast dispersion; corresponding variables have been proposed as reflecting investor overconfidence in stock market studies).

2 The Forward Discount Puzzle

Under the assumptions of investor risk neutrality and rational expectations, uncovered interest parity (UIP) implies that the forward exchange rate, f_t, should equal the rationally expected future exchange rate, s_{t+1}; equivalently, the forward discount $f_t - s_t$ should be an unbiased forecast of the subsequent exchange rate depreciation, $s_{t+1} - s_t$. (Consistent with past literature, we ignore empirically minor nonlinearity effects associated with Jensen's Inequality.) However, the forward discount often *negatively* forecasts subsequent exchange rate depreciations. In the regression

$$\Delta s_{t+1} = \alpha + \beta(f_t - s_t) + u_{t+1} \tag{1}$$

of the change in the log spot exchange rate, $\triangle s$, on the forward discount $f_t - s_t$, the estimate of β using weekly or monthly data is usually significantly less than one, the average estimate of β across some 75 published estimates surveyed by Froot and Thaler (1990) is -0.88. A negative coefficient is in sharp contrast to the coefficient of $+1$ implied by unbiasedness of forward rates.[3]

The regression coefficients on the forward discount switch from negative to positive at long horizons (see, e.g., the evidence from Meredith and Chinn (2004) using 10-year forward exchange rates). Thus, long-term exchange rate changes are on average positively forecasted by the interest differentials of long-term bonds, but the estimated regression slope coefficient is still often less than one.

[3] An equivalent specification for the forward discount regression (1) is

$$s_{t+1} - f_t = a + b(f_t - s_t) + u_{t+1}, \tag{2}$$

with $a = \alpha$, and $b = \beta - 1$ (so that a negative coefficient β in regression (1) corresponds to $b < -1$ in regression (2)). The left-hand side of (2) is the return from purchasing the one-period forward contract. Results from regression (2) are, not surprisingly, consistent with those for (1); see, e.g., Bekaert and Hodrick (1992).

3 The Basic Idea

Existing models of overconfidence in securities markets imply that returns are predictable based on current market prices and fundamental measures. We review the intuition behind such predictability, and contrast it with the intuition developed here, which reflects the monetary aspect of the forward discount puzzle. We will show that the intuition underyling previous overconfidence models can explain why the forward discount regression yields slope coefficients less than one, but that the distinctive aspects of the foreign exchange setting explains why a negative regression coefficient (the forward discount puzzle) is possible.

Evidence of long-term stock market return reversals (e.g., DeBondt and Thaler, 1985), and that market-to-book and other price/fundamental ratios negatively predict future stock returns (e.g., Rosenberg, Reid and Lanstein, 1985) has often been interpreted as representing market overreaction. Expressed logarithmically, there is a negative slope coefficient in the regression

$$r_{t+1} = \Delta m_{t+1} = \alpha + \beta(m_t - b_t) + v_{t+1}, \tag{3}$$

where m and b are the log market value and book value, respectively, and r_{t+1} is the subsequent stock return. There is much debate about whether the negative coefficient reflects a purely rational risk premium (e.g., Fama and French, 1993) or imperfectly rational beliefs as well (as implied, e.g., by the model of Daniel, Hirshleifer and Subrahmanyam, 2001). The psychological interpretation is that low price/fundamental ratios indicate that the market is too pessimistic about a stock's prospects, pushing the stock price too low. The low price tends to correct, causing a positive average return. The lower the price/fundamental ratios, the higher the average subsequent returns.

Let us define the terminal book value b_{t+1} as the cash flow generated by the firm at the terminal date $t+1$. Then the current book and market values contain distinct information about the terminal value. The current book value reflects mainly past or current cash flows (and whatever this implies on average for future cash flows). The current market value contains additional information about investors' expectations, but is vulnerable to belief biases.

Suppose for simplicity that the current book value b_t is an unbiased predictor of the firm's terminal cash flow based upon the existing information. Then the current market/book ratio $m_t - b_t$ reflects both valid market response and any overreaction to new information about the terminal cash flow. Overreaction to information signals, as proxied by $m_t - b_t$, and its eventual correction can cause a negative slope coefficient in (3). On the other hand, in the regression

$$\triangle b_{t+1} = \alpha + \beta(m_t - b_t) + w_{t+1}, \tag{4}$$

we expect to find $0 \leq \beta < 1$ so long as the market price reflects some meaningful additional information about firm's terminal cash flow beyond that contained in the book value. The favorable information that $m_t > b_t$ predicts a positive change $\triangle b_{t+1}$. Because of overreaction, m_t tends to overpredict b_{t+1}, resulting in a slope coefficient less than 1. In the special case where overconfident investors react to pure noise signals, β should be zero.

In the foreign exchange setting, there is subjective judgment involved in forecasting future inflation, which creates scope for overconfidence.[4] Both the spot exchange rate (like the book value b_t in (1)) and the forward exchange rate (like the market value m_t) contain information about future fundamentals, here the inflation differential. Suppose for the moment that the spot rate, in analogy to the current book value, has little or no average bias, whereas the forward exchange rate, like the current stock market value, is subject to substantial misreaction. Then the forward discount regression (1)

[4]The existence of an active industry selling macroeconomic forecasts is consistent with our assumption that at least some investors believe they can obtain superior information about future inflation. It is not crucial for our purposes whether investors are correct in thinking that they possess superior information about inflation, so long as they overreact.

in Section 2 is similar to the regression (4) in the stock market context. When the forward rate is high relative to the spot rate, the market expects a relatively high inflation differential and exchange rate depreciation.

The coefficient β in (1) can be less than one, because the overconfidence-induced overreaction in investors' expectations causes the forward rate to rise more than the increase in the true expected future spot exchange rate. We illustrate this effect (which is much weaker than the forward discount bias) in Figure 1. The upper half of Figure 1 plots the path of movement for the spot and forward exchange rates from date 0 to date 2, conditional on a positive date-1 signal about date-2 inflation differential. For ease of presentation, we assume that at date 0, the expected future inflation differential is zero so that spot exchange rate s_0 coincides with the forward rate f_0. The expected movement of the spot rate if there is no overreaction is depicted in the line segment from s_0 to s_1^R, and then the segment to the point labelled $E_1^R[s_2]$; the R superscript indicates values under rational belief. If there were no overreaction in the forward rate, the forward rate would be $f_1^R \equiv E_1^R[s_2]$. The overreaction in the forward rate is shown in the steep segment running from s_0 to $f_1 > E_1^R[s_2]$. Under our temporary assumption of no overreaction in the spot rate, the forward discount is the vertical difference $f_1 - s_1^R$. The symmetrical case of a negative signal is in the lower half of the figure.

Comparing the upper and lower halves of the figure, it is evident that a positive forward discount is associated with a higher expected future spot rate than a negative forward discount– i.e., that the β coefficient in the forward discount regression is positive. It is also evident that the coefficient is less than one; the overreaction in the forward rate implies that the variation in the independent variable, $f_1 - s_1^R$, is larger than the average variation in the dependent variable, $E_1^R[s_2] - s_1^R = f_1^R - s_1^R$ by the amount of overreaction $f_1 - f_1^R$.

The greater the importance of overreaction relative to genuine information in the forward rate movement, the lower the β coefficient. As overreaction becomes extreme, so that the date 1 forward rate swings wildly relative to the future spot rate, the regression coefficient approaches zero, but does not become negative. Thus, when there is no overreaction in the spot rate, we do not obtain the forward discount puzzle. In other words, the intuition provided by previous models from stock markets can explain only a softened version of the forward discount puzzle, a coefficient on the forward discount that is less than one. The puzzle remains: why is the coefficient negative?

Our answer relies on a difference between the foreign exchange setting and the stock market setting. Whereas book value is an historically-determined quantity, the spot exchange rate is a current market price, subject to its own misreaction. For example, suppose that investors receive a signal about an increase in the U.S. relative to German inflation. The forward rate (Dollar/DM) rises, incorporating the expected depreciation. The spot rate rises too (and may also overshoot), because investors who expect higher future U.S. inflation are less willing to hold dollars today. In our model, the inflation signal endogenously has a stronger effect on the forward rate than on the current spot rate. Therefore, the forward discount is positively related to overconfidence-induced overreaction in the spot exchange rate and predicts its subsequent correction. This effect can result in a negative slope coefficient.[5] Whether it does so depends upon the balance between the traditional effect (the fact that the forward rate reflects information about future inflation) and the overreaction/correction effect.

This intuition is also illustrated in Figure 1. After a positive private signal (the upper branch of the figure), owing to overreaction, the forward rate rises above the level of the new expected spot rate $E_1^R[s_2]$; the spot rate rises less, because consumption good price levels and spot exchange rates are influenced by a transactions demand

[5]The stock market analogy to this reasoning would be for the market-to-book ratio to negatively predict the future change in the book value over time. No such empirical regularity exists in the stock market, nor is this predicted by models of overconfidence in the stock market. This illustrates a key difference between the two contexts; here the spot rate is a market price which can overreact, whereas in the stock market context book value does not.

for money, not just speculative concerns about future inflation rates; so the forward discount $f_1 - s_1$ is positive.[6] Symmetrically, on the lower branch, the forward rate declines more than the spot rate, so that the discount is negative. At date 2 the overreaction in the spot rate corrects. If the spot overreaction is strong enough, then in the upper branch of the figure in which the forward discount is positive, on average (conditional on date 1 information) the spot rate declines to $E_1^R[s_2]$; and in the lower branch in which the forward discount is negative, on average the spot rate increases to $E_1^R[s_2]$. Thus, the forward discount is a negative predictor of the change in the spot rate.

A common challenge to psychology-based approaches to securities markets anomalies is to explain how irrational investors can have an important effect if there are smart arbitrageurs. If biased investor beliefs distort the forward discount so that $\beta < 1$, then rational investors who buy currencies that offer higher-than-usual interest rates and sell those with lower-than-usual interest rates should earn excess returns. However, Froot and Thaler (1990) provide some calculations of the possible arbitrage profit based on this anomaly, and argue that transactions costs and other practical constraints limit the profitability of such strategies. Even if smart investors on average do better, this is not a riskfree arbitrage opportunity. Hence, we do not expect complete elimination of the forward discount bias.

Several arguments have been made in the literature as to why irrational investors do not necessarily lose in the competition with the rational ones.[7] Even if less sophisticated currency users on average lose relative to a set of smart speculators, less sophisticated individuals will still need to hold money balances, so their money de-

[6]To put this another way, covered interest rate parity (defined near the start of Subsection 4.4) implies that the forward rate differs from the spot rate by the interest rate differential, which reflects expectations of an inflation differential; so when news arrives indicating that such a differential is likely to be large, the forward discount widens.

[7]Reasons that imperfectly rational investors may earn high expected profits and/or remain important in the long run include a possible greater willingness of overconfident investors to bear risk or to exploit information aggressively, limited investment horizons of the arbitrageurs, wealth reshuffling across generations, and the existence of market frictions (see Hirshleifer (2001) for a discussion of these issues).

mands will still play a role in determining equilibrium price levels and therefore spot and forward exchange rates.

4 The Model

The currencies of Countries A and B can be exchanged costlessly. Each country's products can only be purchased using its home currency.

4.1 Decisionmakers

There are two groups of decisionmakers. One consists of individuals who receive information signals about future money growth, and are overconfident in the sense that they overestimate the precision of their signal. The other group consists of individuals who do not receive signals and are thus rational (not overconfident). The inclusion of non-signal-receiving individuals allows us to interpret the signals received by informed individuals as 'private', and therefore overconfidence-inducing (Daniel, Hirshleifer and Subrahmanyam, 1998). For modelling convenience, we assume that the overconfident individuals are risk neutral, while the rational individuals are risk averse. This implies that prices are determined solely by the overconfident investors.[8]

[8] In a more general setting where all investors are risk averse, prices reflect a weighted average of the beliefs of different investors, and therefore still reflect overconfidence; see, e.g., Daniel, Hirshleifer and Subrahmanyam (2001). Similar results to those derived here would apply in such a setting. Similar results also would apply if there are some risk averse, fully rational "arbitrageurs" who received information signals. Only when the fraction of individuals who are fully rational approaches one would the effects described in this paper vanish. In the more general settings, risk is priced as well; such risk effects are not essential to our argument.

4.2 Money Markets

We assume a typical Cagan money demand function in logarithmic form,

$$\begin{aligned} m_t^d - p_t &= c - \alpha \pi_{t+1}^e \\ m_t^{d*} - p_t^* &= c^* - \alpha^* \pi_{t+1}^{e*}, \end{aligned}$$

where m_t^d and p_t are respectively the log money demand and price level in country A at date t, $\pi_{t+1} = p_{t+1} - p_t$ is the realized inflation from date t to $t+1$ in country A, and $\pi_{t+1}^e \equiv \mathrm{E}_t[\pi_{t+1}]$ is the expected inflation rate from t to $t+1$ period as perceived at date t by money-holding individuals. An asterisk denotes a country B (foreign country) variable. The constants c and c^* represent the effect of the output and the real interest rate which are assumed to be fixed in the short run. Constant parameter $\alpha > 0$ measures the sensitivity of money demand to inflation expectations. For simplicity, we assume that $\alpha = \alpha^*$.

The log money supplies in both countries, m_t and m_t^*, are exogenously determined by the monetary authorities. We can view the exogenous money supplies as nominal wealth endowments in each period to the individuals. We define money growth rates in countries A and B from date t to $t+1$, respectively as

$$\begin{aligned} \mu_t &= m_t - m_{t-1} \\ \mu_t^* &= m_t^* - m_{t-1}^*. \end{aligned}$$

The money growth differential between the two countries at date t, $\bar{\mu}_t \equiv \mu_t - \mu_t^*$, is the economic fundamental in our model. The money markets are continuously equilibrated. Money market equilibrium requires that $m_t^d = m_t$ and $m_t^{d*} = m_t^*$ for all t.

We define the *realized* inflation differential across the two countries on date t as $\bar{\pi}_t \equiv \pi_t - \pi_t^*$, and the *expected* next period inflation differential as $\bar{\pi}_t^e \equiv \mathrm{E}_t[\bar{\pi}_{t+1}]$. The

Cagan demand function implies that the date t realized inflation differential satisfies

$$\bar{\pi}_t = \triangle p_t - \triangle p_t^* = \bar{\mu}_t + \alpha \triangle \bar{\pi}_t^e, \tag{5}$$

where $\triangle \bar{\pi}_t^e \equiv \bar{\pi}_t^e - \bar{\pi}_{t-1}^e$ is the *revision* of the expected inflation differential on date t as new information arrives. Equation (5) implies that the current realized inflation differential responds to any changes in expectations of future inflation. If informed investors expect a higher inflation differential tomorrow, the inflation differential starts to widen today. How much price levels move today depends on the sensitivity of price indices to changes in inflation expectations, as reflected in α.

4.3 Spot Exchange Rates and PPP

The date t spot exchange rate depreciation is defined as $\triangle s_t \equiv s_t - s_{t-1}$, where s_t is the date t log spot exchange rate (the price of currency B in terms of currency A). We assume that the dynamics of the spot rate depreciation is

$$\triangle s_t = \theta \bar{\pi}_t - \kappa(\triangle s_{t-1} - \bar{\pi}_{t-1}). \tag{6}$$

In this dynamics, relative purchasing power parity (PPP) holds exactly when $\theta = 1$. When $\theta > 1$, the spot rate depreciation overshoots the realized inflation differential. $\kappa > 0$ reflects a partial correction of previous spot rate overshooting and a gradual convergence towards PPP in the long run.

The above dynamics is motivated by two sets of stylized facts in the international finance literature. First, spot exchange rates are much more variable than commodity price levels. When researchers regress $\triangle s$ on $\bar{\pi}$, the estimated slope coefficient often significantly exceeds unity (see, e.g., Krugman, 1978; Frenkel, 1981). Some economists argue that such violations of PPP reflect different spped of adjustment of exchange rates and of consumer price indices to economic shocks (e.g., Dornbusch, 1976). It has also been suggested that exchange rates, like the prices of other durable assets, are much more forward-looking and sensitive to market expectations than national commodity price levels (e.g., Frenkel, 1981). Real barriers to market integration have also been shown to be important empirically in explaining the failure of the law of one price (e.g., Engel and Rogers, 2001).

Second, although PPP is violated in the short run, empirical research has documented mean-reversion in PPP deviations, as well as long-run trends in nominal and real exchange rates that are consistent with PPP.[9] The dynamics described in (6) captures both short term overshooting in exchange rates and long term mean-reversion in deviations of exchange rates from PPP.

4.4 The Bond Market and the Forward Discount

Investors trade bonds denominated in currency A and currency B. The nominal returns on one-period bonds, i.e., the nominal interest rates, follow the Fisher equation:

$$\begin{aligned} i_t &= r_t + \pi_t^e \\ i_t^* &= r_t^* + \pi_t^{e*}, \end{aligned}$$

where r_t and r_t^* are real rates of return on the bonds of countries A and B, respectively. For simplicity, and to focus on the market's ability to process information about future inflation, we assume that $r_t = r_t^*$. Thus, the determination of real bond returns is exogenous to the model.

Covered interest rate parity, a standard arbitrage condition in the forward exchange market, implies that

$$d_t \equiv f_t - s_t = b_t^* - b_t = i_t - i_t^* = \bar{\pi}_t^e, \tag{7}$$

where b_t and b_t^* are the log prices for bonds denominated in currencies A and B,

[9] See, e.g., Frankel (1986); Edison (1987); Glen (1992); Frankel and Rose (1995); Mark and Choi (1997).

respectively. The one-period forward discount is equal to the nominal interest rate differential, which is also the expected inflation differential in our model.

4.5 Information Structure and Signals

Without loss of generality, we assume that on the initial date 0, $\triangle s_0 = \bar{\pi}_0 = \bar{\mu}_0$, where $\bar{\mu}_0$, a constant, can be interpreted heuristically as the long-run equilibrium money growth differential between countries A and B. The realization of the money growth differential on each date after date 0 is

$$\begin{aligned} \bar{\mu}_1 &= \bar{\mu}_0 + \eta_1 \\ \bar{\mu}_t &= \bar{\mu}_0 + \eta_t + u, \quad t \geq 2, \end{aligned} \tag{8}$$

where η_t is a white noise term in the money growth differential process. It is independent over time, and is independent of u, a persistent shock in the money growth differential process which arrives on date 2. We assume $u \sim N(0, V_u)$.

Informed individuals revise their inflation expectations at date 1 based upon a noisy signal about u that takes the form

$$\sigma = u + \epsilon, \tag{9}$$

where ϵ, the signal noise, is distributed as $N(0, V_\epsilon)$. We assume that the informed individuals overestimate the precision of their 'private' signals. In other words, they believe that the variance of the signal noise is lower than the true level: $V_\epsilon^C < V_\epsilon$, where a superscript C denotes an overconfident perception. Let the information precision be $\nu_\epsilon \equiv 1/V_\epsilon$ and $\nu_u \equiv 1/V_u$. Then $\nu_\epsilon^C \equiv 1/V_\epsilon^C > \nu_\epsilon$, which implies that the overconfident investors take the noisy signal as more informative than it actually is, and tend to overreact to it. On date 2, we assume that the shock u in the money growth differential is realized, and overconfident investors correct their date 1 expectation errors.[10]

[10]The assumption that u is realized on date 2 is made for simplicity of presentation. Qualitatively

5 The Forward Discount Bias

In this section, we derive the date 1 forward discount and the date 2 spot exchange rate change. We show that overconfidence-induced overreaction to the money growth news and its subsequent correction can explain the negative relationship between the forward discount and the future exchange rate movement.

To facilitate the analysis, we introduce the following lemma that summarizes the relationship between the key variables in our model (The proof is provided in the Appendix). $\mathrm{E}[-]$ denotes expectations taken with respect to the beliefs of the overconfident investors, and $\mathrm{E}^R[-]$ denotes rational expectations.

Lemma 1 *Let $\triangle s_t$ be the change in the nominal spot exchange rate, d_t be the one-period forward discount, $y_t \equiv \triangle s_t - \bar{\pi}_t$ be the change in the real exchange rate, and $z_t \equiv \triangle \bar{\pi}^e_t$ be the change in expected inflation differential. Under the model assumptions of Subsections 4.2-4.4, these variables are related to the fundamental variable $\bar{\mu}_t$, the money growth differential, as follows:*

$$\begin{aligned} \triangle s_t &= \bar{\mu}_t + \alpha z_t + y_t, && (10)\\ z_t &= \mathrm{E}_t[\bar{\mu}_{t+1}] - \mathrm{E}_{t-1}[\bar{\mu}_t] + \alpha \mathrm{E}_t[z_{t+1}] - \alpha \mathrm{E}_{t-1}[z_t], && (11)\\ y_t &= (\theta - 1)\sum_{n=0}^{t-1}(-\kappa)^n \bar{\pi}_{t-n}, \quad \text{and} && (12)\\ d_{t-1} &= \mathrm{E}_{t-1}[\bar{\mu}_t] + \alpha \mathrm{E}_{t-1}[z_t], && (13) \end{aligned}$$

where

$$\mathrm{E}_{t-1}[z_t] = \mathrm{E}_{t-1}[\bar{\mu}_{t+1} - \bar{\mu}_t] + \sum_{k=0}^{\infty} \frac{\alpha^k}{(1+\alpha)^k} \mathrm{E}_{t-1}[\bar{\mu}_{t+k+2} - 2\bar{\mu}_{t+k+1} + \bar{\mu}_{t+k}]. \qquad (14)$$

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similar results apply if noisy signals about u are observed after date 1.

5.1 Expectations and Date 1 Spot and Forward Exchange Rates

After receiving the date 1 signal, informed investors update their expectations about the future money growth differential in a Bayesian fashion. Their expectations, however, are subject to the overconfidence bias. We define

$$\lambda^C \equiv \frac{\nu_\epsilon^C}{\nu_u + \nu_\epsilon^C}, \quad \lambda^R \equiv \frac{\nu_\epsilon}{\nu_u + \nu_\epsilon} \quad \text{and} \quad \gamma \equiv \frac{\lambda^C - \lambda^R}{\lambda^C}, \tag{15}$$

where the superscript R denotes a fully rational perception. λ is the signal-to-noise ratio, which measures how sensitive the market expectation is to the inflation signal. Overconfidence implies that $\lambda^C > \lambda^R$, and thus $0 < \gamma < 1$. γ is monotonically increasing in $\nu_\epsilon^C/\nu_\epsilon$: the higher the overconfident investors's perceived signal precision ν_ϵ^C is relative to the truth ν_ϵ, the larger is the γ. Thus, γ measures the degree of overconfidence.

By (9), the overconfident individuals' expectation of future money growth differential on date 1 conditional on the signal realization σ is

$$\mathrm{E}_1[\bar{\mu}_2|\sigma] = \bar{\mu}_0 + \lambda^C\sigma. \tag{16}$$

By the definition of γ in (15), the fully rational conditional expectation is

$$\mathrm{E}_1^R[\bar{\mu}_2|\sigma] = \bar{\mu}_0 + \lambda^R\sigma = \bar{\mu}_0 + (1-\gamma)\lambda^C\sigma.$$

The difference between the two expectations is the overreaction in individuals' perceptions.

Applying equation (16) and Lemma 1 to the information structure in Section 4.5, we have

$$\begin{aligned}\bar{\pi}_1 &= \bar{\mu}_1 + \alpha\lambda^C\sigma \\ \triangle s_1 &= \theta\bar{\pi}_1 = \theta\bar{\mu}_1 + \theta\alpha\lambda^C\sigma \qquad (17) \\ d_1 &= \mathrm{E}_1[\bar{\pi}_2] = \mathrm{E}_1[\bar{\mu}_2] = \bar{\mu}_0 + \lambda^C\sigma. \qquad (18)\end{aligned}$$

We can see that following a positive signal about country A's future money growth rate ($\sigma > 0$), the date 1 inflation differential, the spot exchange rate, and the forward discount all move upward. All three variables contain an element of overconfidence-induced overreaction, as indicated by λ^C.

The intuition behind these equations is as follows. The signal about future money growth differential is informative about the relative value of currency A to currency B. In anticipation of an increase in inflation in country A, an informed individual will tend to hold less currency A, which leads to an immediate depreciation of currency A in the spot market. The more overconfident investors are about their information signal (larger λ^C), and the more volatile the exchange rate is relative to the inflation differential (larger θ), the larger the overreaction in the date 1 spot rate. At the same time, the expectation of future higher inflation in country A also makes currency A to be sold forward at a discount. Since the forward rate is more forward-looking and sensitive to expectations, the forward rate rises even more than the spot rate. Therefore, the spread between the forward and spot rates (i.e., the forward discount) contains the overconfidence-induced overreaction in the spot exchange rate.

5.2 Date 2 Exchange Rate Depreciation

A shock u to the money growth differential is realized on date 2 and persists to date 3. On date 2, the expected money growth differential over the next period is

$$\mathrm{E}_2[\bar{\mu}_3] = \bar{\mu}_0 + u. \qquad (19)$$

Let

$$\delta \equiv \mathrm{E}^R[u|\sigma] - u = \lambda^R \sigma - u \tag{20}$$

be the error in the rational expectations forecast of money growth differential, which is orthogonal to the date 1 information set, $\mathrm{E}^R[\delta|\sigma] = 0$.

Substituting equations (19) and (20) into (5), the date 2 inflation differential is

$$\begin{aligned} \bar{\pi}_2 &= \bar{\mu}_2 + \alpha(\mathrm{E}_2[\bar{\mu}_3] - \mathrm{E}_1[\bar{\mu}_2]) \\ &= (\bar{\mu}_0 + u) - \alpha(\delta + \gamma\lambda^C\sigma) + \eta_2. \end{aligned} \tag{21}$$

The first term on the right hand side of (21) is the new equilibrium money growth differential. The second term is a correction of the inflation expectation error. The expectation error contains two elements: the surprise relative to the rational expectation (δ) and the correction of the overconfidence-induced date 1 overreaction ($\gamma\lambda^C = \lambda^C - \lambda^R$).

Applying Lemma 1 on date 2, we can derive the date 2 exchange rate depreciation $\triangle s_2$. The following proposition states the relationship between the date 2 exchange rate depreciation and the date 1 forward discount.

Proposition 1 *The date 2 spot exchange rate depreciation is a stochastically linear function of the long-run money growth differential* $\bar{\mu}_0$ *and the forward discount* d_1*. Specifically,*

$$\triangle s_2 = \beta_0 \bar{\mu}_0 + \beta_1 d_1 + v_2, \tag{22}$$

where

$$\begin{aligned} \beta_0 &= (1+\alpha)\gamma\theta + \kappa(\alpha - 1)(\theta - 1), \\ \beta_1 &= 1 - (1+\alpha)\gamma\theta + (1 - \alpha\kappa)(\theta - 1), \\ v_2 &= \theta\eta_2 - \kappa(\theta - 1)\eta_1 - (1+\alpha)\theta\delta, \end{aligned} \tag{23}$$

with $\mathrm{E}^R[v_2] = 0$ *and* $\mathrm{cov}(d_1, v_2) = 0$.

Proposition 1 is a key result of our paper. It shows that β_1, the slope coefficient on the forward discount, can be decomposed into three terms. The first term of β_1 is unity, which represents the conventional effect (uncovered interest parity). The second term reflects investor overconfidence, and the last term reflects deviations from PPP. We consider the implications of Proposition 1 in several special cases with different combinations of θ and γ.

Case 1: $\gamma = 0$, and $\theta = 1$ (no overconfidence, no deviations from PPP). In this case, $\beta_1 = 1$, $\beta_0 = 0$. The date 2 exchange rate depreciation is then

$$\triangle s_2 = d_1 + v_2 = \bar{\mu}_2 - \alpha\delta = \bar{\pi}_2.$$

The date 1 forward discount is an unbiased predictor of the date 2 exchange rate depreciation. Uncovered interest rate parity holds.

Case 2: $\gamma > 0$, and $\theta = 1$ (overconfidence, no deviations from PPP). In this case, $\beta_1 = 1 - (1 + \alpha)\gamma$ is less than unity. Furthermore, if

$$\gamma > \gamma_0 = \frac{1}{1+\alpha},$$

then $\beta_1 < 0$. The more overconfident investors are, the more negative the relationship between the forward discount and the subsequent exchange rate depreciation.

Case 3: $\gamma = 0$, and $\theta > 1$ (no overconfidence, deviations from PPP). In this case, $\beta_1 = 1 + (1 - \alpha\kappa)(\theta - 1)$, which will be negative if and only if

$$\alpha\kappa > 1 \text{ and } \; \theta > 1 + \frac{1}{\alpha\kappa - 1}.$$

Empirical studies on Cagan's money demand function and the short- and long-run validity of PPP suggest that the condition above is inconsistent with the data. The estimated values of $\alpha\kappa$ are generally less than one, since κ is small due to high persistence of real exchange rate. Even if $\alpha\kappa > 1$, e.g., $\alpha\kappa = 1.5$, we need $\theta > 3$ to have a negative β_1. Such high θ, however, is not observed in the empirical research.[11] In other words, deviations from PPP alone do not explain the forward discount puzzle.

Case 4: $\gamma > 0$, and $\theta > 1$ (overconfidence, deviations from PPP). This case displays the interactions between the deviation from perfect investor rationality and the deviation from relative purchasing power parity in explaining the forward discount puzzle. $\beta_1 < 0$ if

$$\gamma > \gamma_0' = \frac{1 + (1 - \alpha\kappa)(\theta - 1)}{(1 + \alpha)\theta}.$$

Compared with Case 2 ($\theta = 1$), $\gamma_0' < \gamma_0$, which implies that $\theta > 1$ magnifies the effect of overconfidence and lowers the required level of overconfidence to get a negative regression coefficient. Compared with Case 3 ($\gamma = 0$), a positive γ always leads to smaller or more negative β_1.

To see the intuition for Cases 1-4, suppose first as in Case 2 that there is overconfidence and no deviations from PPP. In this case there are two opposing effects operating upon the dependent variable in regression (1), the exchange rate depreciation. On the one hand, a higher inflation differential between countries A and B, when realized, depreciates currency A ($\triangle s > 0$). This is the standard, fully rational effect. On the other hand, the overshooting of the spot rate due to initial overreaction eventually gets corrected, which promotes an appreciation of the dollar ($\triangle s < 0$). This is the overreaction-correction effect. The sign of $\triangle s$, and therefore the sign of β, depends on which effect dominates. If an information signal is pure noise, the overreaction-correction effect must dominate, leading to a negative slope coefficient, the forward discount anomaly. Even for meaningful signals, greater overconfidence increases overreaction in the spot rate, thus strengthening the overreaction-correction effect. When the overreaction-correction effect dominates, a negative relation between

[11] Cavallo, Kisselev, Perri and Roubini (2005) estimate real exchange rate overshooting and find the amount of overshooting to be below 50% except for two countries. The largest overshooting is about 150% in their sample.

the forward discount and the subsequent exchange rate changes results.

Thus, the short-term forward discount negatively forecasts subsequent exchange rate changes because the forward discount reflects overreaction in the spot exchange rate and predicts its later correction. There are two possible drivers of spot rate overreaction: overconfidence and deviations from PPP (see (17)). The date 1 spot rate overreaction and its subsequent correction are the strongest when both effects are present, leading to a more negative β_1. However, the spot rate overshooting induced by investor overconfidence and that induced by the PPP deviation have different effects on β_1. Case 3 shows that PPP deviations alone can not explain the forward discount bias. This is because when there is no investor overconfidence, the forward discount contains no overreaction (see (18)), and thus can not predict the subsequent reversal of the spot rate.

The following numerical example illustrates some possible values of β_1, given the values of α, κ, γ and θ. Based on many empirical estimates of Cagan's model (e.g., Cagan, 1956; Barro, 1970; Goodfriend, 1982; Phylaktis and Taylor, 1993), we set $\alpha = 4$. Rogoff (1996) concludes that the consensus view on the half-life of real exchange rate is between three and five years. We choose $\kappa = 0.1733$ which gives a half life of four years. We vary the values of γ and θ to show the effects of overconfidence and PPP deviations on β_1.

$\gamma \backslash \theta$	1.0	1.25	1.33	1.5
0.00	1.00	1.08	1.10	1.15
0.25	-0.25	-0.49	-0.56	-0.72
0.33	-0.67	-1.01	-1.12	-1.35
0.5	-1.50	-2.05	-2.23	-2.60

Table 1: The Effects of Overconfidence and PPP Deviations on β_1

Table 1 illustrates that the greater is investor overconfidence, the more pronounced is the forward discount bias. By (23),

$$\frac{\partial \beta_1}{\partial \gamma} = -(1+\alpha)\theta < 0.$$

We summarize the discussion above in the following proposition.

Proposition 2 *When investor overconfidence is sufficiently strong, the slope coefficient on the forward discount in a time series regression (equation (1)) is negative. The greater the degree of investor overconfidence, the more pronounced the forward discount bias.*

Further Empirical Implications

An empirical implication of Proposition 2 is that the magnitude of the forward discount bias will change over time as the level of investor overconfidence shifts. The forward discount bias will be more pronounced in periods in which investors are more overconfident. Previous theoretical research has found that the belief overreactions caused by overconfidence increase volume of trade, return volatility, and cross-firm valuation dispersion; there is empirical evidence supporting such effects.[12] Similar effects apply in our setting as well; overconfident foreign exchange investors will trade more with rational investors, and induce greater price overreactions, when overconfidence is greater. We therefore predict a positive time-series relationship between the strength of the forward discount anomaly and abnormal trading volume, excess exchange rate volatility (relative to the volatility of the money growth rate or the inflation rate), and the dispersion of exchange rate forecasts in the foreign exchange markets.

Our model also has implications about the relation between the forward discount and investors' expectational forecast error for future exchange rate depreciation. By

[12]E.g., Odean (1998); Gervais and Odean (2001); Glaser and Weber (2003); Statman, Thorley and Vorkink (2005); Jiang (2005).

Proposition 1, the expected forecast error made by investors is

$$E[\Delta s_2|\sigma] - \Delta s_2 = -(1+\alpha)\gamma\theta\bar{\mu}_0 + (1+\alpha)\gamma\theta d_1 + \theta[(1+\alpha)\delta - \eta_2].$$

It follows immediately that:

Corollary 1 *When investors are overconfident ($\gamma > 0$), their prediction error is positively correlated with the forward discount.*

This is because overconfidence causes investors' expectations to overreact, and the forward discount reflects such overreaction. This implication is supported by the empirical findings in Froot and Frankel (1989) and Frankel and Chinn (1993). For example, Froot and Frankel (1989) run the regression

$$\Delta\hat{s}^e_{t+1} - \Delta s_{t+1} = \alpha_1 + \hat{\beta}_1(f_t - s_t) + v_{t+1},$$

where Δs_{t+1} is the realized change in the spot exchange rate between date t and $t+1$, and $\Delta\hat{s}^e_{t+1} = s^e_{t+1} - s_t$, with s^e_{t+1} being investors' expectation on date t of the date $t+1$ spot exchange rate (computed using survey data). They find (see their Table V) that $\hat{\beta}_1$ is significantly greater than zero, a finding that is robust across different forecast horizons and different survey samples.

5.3 Relative Predictive Power of Different Spot Rate Predictors

We have shown that investor overconfidence implies that the forward discount can negatively predict spot exchange rate changes, because the forward discount reflects overreaction in the spot rate and predicts its subsequent correction. However, similar reasoning (and inspection of Figure 1) implies that other variables that reflect mispricing, such as the inflation differential, the forward rate, the spot rate, the latest change in the spot or forward rate, can also predict exchange rate changes. In this section, we show that when investors are overconfident, the forward discount is a stronger predictor of the subsequent spot rate reversal than these alternatives. To isolate the influence of the overconfidence-induced overreaction, we set $\theta = 1$.

Proposition 3 *If purchasing power parity holds ($\theta = 1$), then the forward discount is the strongest predictor of subsequent exchange rate changes (highest R^2) among the possible alternatives suggested by our model (the forward discount, the inflation differential, the forward rate, the spot rate, the latest change in the forward rate, and the latest change in the spot rate).*

The proof of this proposition is provided in the appendix. Intuitively, the innovation in the money growth differential at date 1, η_1, is independent of the future money growth differential and unrelated to the private signal σ. Thus, for the purpose of predicting future spot exchange rates, η_1 is noise: it is unrelated to the overconfidence-induced mispricing in the spot rate. The money growth surprise η_1 is reflected in the realized inflation differential, $\bar{\pi}_1$, spot rate, s_1, forward rate, f_1, and latest changes in these rates, $\triangle s_1$ and $\triangle f_1$, but is differenced out from the forward discount, $f_1 - s_1$. Thus, the forward discount, as a purer measure of the spot rate overreaction, has more predictive power in forecasting the future correction of such overreaction.

5.4 Long-Horizon Forward Discount Regressions

In Section 5.2, we show that in our model, there is usually a negative relationship between the one-period forward discount and the subsequent one-period change in exchange rate. We now examine the relation between the forward discount and the future spot rate change in a longer-horizon regression. Specifically, in a regression of the two-period change in spot exchange rate $s_3 - s_1$ on the two-period forward discount $d_{1,3} \equiv f_{1,3} - s_1$, where $f_{1,3}$ is the forward exchange rate for a two-period forward contract, we examine whether the slope coefficient will be larger or smaller than that in (22). Our findings are summarized in the following proposition (proof in the appendix).

Proposition 4 *In the two-period forward discount regression,*

$$s_3 - s_1 = \beta_0' \overline{\mu}_0 + \beta_1' d_{1,3} + v_3,$$

the coefficient on the forward discount is

$$\beta_1' = 1 + 0.5(\theta - 1) - 0.5[(2+\alpha)\theta - (1+\alpha)\kappa(\theta - 1)]\gamma + 0.5[1 - \kappa - \alpha\kappa(1-\kappa)](\theta - 1).$$

The difference between the slope coefficient β_1 in the one-period forward discount regression and the slope coefficient β_1' in the two-period forward discount regression is

$$\beta_1 - \beta_1' = -0.5\{[\alpha\theta + (1+\alpha)\kappa(\theta-1)]\gamma + \kappa(\alpha + \alpha\kappa - 1)(\theta - 1)\},$$

so for a sufficiently high level of overconfidence, the slope coefficient in the one-period forward discount regression is more negative than the slope coefficient in the two-period forward discount regression, i.e., $\beta_1 < \beta_1'$, if

$$\gamma > \frac{\kappa(1 - \alpha - \alpha\kappa)(\theta - 1)}{\alpha\theta + \kappa(1+\alpha)(\theta - 1)}.$$

If $\theta = 1$, then $\beta_1 < \beta_1'$ holds so long as there is overconfidence. When $\theta > 1$, for empirical relevant values of α ($\alpha > 1$) based on estimates of the Cagan money demand function, we still have $\beta_1 < \beta_1'$ for all level of overconfidence. The following table shows some possible values for β_1', using the same parameter values $\alpha = 4$, $\kappa = 0.1733$, and the same combinations of θ and γ as in Table 1.

$\gamma\backslash\ \theta$	1.0	1.25	1.33	1.5
0.00	1.00	1.16	1.21	1.31
0.25	0.25	0.25	0.25	0.24
0.33	0.00	-0.06	-0.08	-0.11
0.5	-0.50	-0.66	-0.72	-0.83

Table 2: The Effects of Overconfidence and PPP Deviations on β_1' in a Two-Period Regression

Proposition 4 indicates that when investors are overconfident, the two-period forward discount is still a biased predictor of the subsequent two-period exchange rate depreciation. However, the forward discount bias becomes less pronounced in the two-period regression than in the one-period regression. This implication of our model is consistent with the empirical findings of Gourinchas and Tornell (2004) (using 3-month, 6-month and 12-month forward discounts), and Meredith and Chinn (2004) (using 5 and 10 year forward discounts).

Intuitively, the sign of the slope coefficient in the forward discount regression depends on the relative strength of two opposing effects: the conventional effect (UIP) and the overreaction-correction effect. At short horizons, the overreaction-correction effect tends to dominate. However, much of the correction of misperceptions will tend to occur in the short run as news about inflation arrives. In contrast, there are fairly objective and well understood differences in countries' expected money growth rates that can persist over very long periods of time. So at longer horizons, the traditional effect will tend to dominate.

The same intuition can be applied to the relationship between long-run average exchange rate depreciations and the long-run average forward discount. Substituting (18) into (22), and taking the unconditional expectation of both sides, we have

$$\mathrm{E}[\triangle s_2] = (\beta_0 + \beta_1)\bar{\mu}_0 = \mathrm{E}[d_1]. \tag{24}$$

Equation (24) implies that although the short-term forward discount negatively predicts the subsequent exchange rate depreciation, the long-term average forward discount correctly predicts the long-term average future exchange rate depreciation. In other words, our model implies that one can earn excess returns by holding bonds from countries whose nominal interest rates are *temporarily higher than usual* relative to the interest rate in other countries, but one does not earn more by simply holding bonds from countries with higher interest rates than others. This is consistent with the empirical findings that countries with steadily higher interest rates (than that in the U.S.) have steady currency depreciations (against the U.S. dollar), as predicted by UIP (see Cochrane, 1999). Again, our model suggests that this is because the conventional effect dominates the overreaction-correction effect in the long run.

5.5 Cross-Sectional Implications

So far we have considered time-series forward discount regressions. In this section, we turn to the implications of our model for cross-sectional forward discount regressions. Consider the exchange rates between a fixed home country and N foreign countries, denoted by i, $i = 1, 2, ..., N$. As before, the exchange rates are defined as the prices of foreign countries' currencies in units of the home country's currency. We make the same assumptions as in the basic model (including the same parameters) for all country pairs. We now add a superscript i to denote a given country pair. Then equation (22) applies to each country pair with the same β_0 and β_1 coefficients as given in Proposition 1:

$$\triangle s_2^i = \beta_0 \bar{\mu}_0^i + \beta_1 d_1^i + v_2^i, \qquad i = 1, 2, ..., N. \tag{25}$$

There is, however, one difference between (25) and (22). $\bar{\mu}_0$ is a constant term in time series regression (22), but is a random variable in (25), because different country pairs have different average money growth differentials. This implies that the cross-sectional variation in $\bar{\mu}_0^i$ will help explain part of the cross-sectional variation in future exchange rate depreciations across different country pairs. Furthermore, $\bar{\mu}_0^i$ is positively correlated with d_1^i in the cross-section ($d_1^i = \bar{\mu}_0^i + \lambda^C \sigma^i$), which implies that $\bar{\mu}_0^i$ would affect the slope coefficient b_1 in the cross-sectional regression

$$\triangle s_2^i = b_0 + b_1 d_1^i + v_2^i, \qquad i = 1, 2, ..., N, \tag{26}$$

where b_0 is a constant. In contrast, in the time-series regression (1) for a specific pair of countries, $\overline{\mu}_0$ is the same over time. It only affects the future exchange rate depreciation through the constant term in the regression, and has no influence on the slope coefficient.

To compute b_1 in (26), we first project $\bar{\mu}_0^i$ onto d_1^i. Assume that the $\bar{\mu}_0^i$'s are drawn from a normal distribution $\bar{\mu}_0^i \sim N(0, V_{\bar{\mu}})$. Then

$$d_1^i = \bar{\mu}_0^i + \lambda^C \sigma^i \sim N(0, V_{\bar{\mu}} + (\lambda^C)^2 V_u).$$

Since $\bar{\mu}_0^i$ and σ^i are independent, we have

$$\mathrm{E}[\bar{\mu}_0^i | d_1^i] = \rho d_1^i, \tag{27}$$

where

$$\rho \equiv \frac{V_{\bar{\mu}}}{V_{\bar{\mu}} + (\lambda^C)^2 V_u}.$$

The parameter ρ is between 0 and 1, and decreases with investor overconfidence (measured by λ^C). By equations (25) and (27), the slope coefficient b_1 in cross-sectional forward discount regression is

$$b_1 = \rho \beta_0 + \beta_1 = \rho + (1 - \rho)\beta_1.$$

Thus, b_1 is a weighted average of the time-series forward discount regression coefficient

β_1 given in (23) and the regression coefficient implied by UIP (i.e., unity). Therefore,

$$\beta_1 < b_1 < 1;$$

the forward discount bias remains in the cross-sectional regression setting, but is less pronounced than in time-series regressions. However, like β_1, b_1 also decreases as λ^C increases, which means that greater investor overconfidence increases the magnitude of the forward discount bias, so the cross-sectional and time series regressions are similar in this respect. We summarize these findings in Proposition 5.

Proposition 5 1. *In both time-series and cross-sectional forward discount regressions, the degree of forward discount bias increases in the level of investor overconfidence.*

2. *Given the level of overconfidence, the forward discount bias is less pronounced in a cross-sectional regression than it is in a time-series regression.*

The intuition for the relative weakening of the forward discount bias in cross-sectional regressions is related to the different roles of the innovation component (u) and the predictable long run component ($\bar{\mu}_0$) of the money growth differential. The realization of u implies a change in the money growth differential process (and also in the inflation differential process in our model) from its past trend. The ex ante average level of the money growth differential $\bar{\mu}_0$ is publicly known and thus is not a matter for overconfident judgment. The overconfidence bias therefore implies greater overreaction to changes than to the more tangible long-run levels. For example, if the money growth differential across the two countries has been steady around 3% for the past 10 years, no one would overreact to the fact that it is at 3% rather than the global mean looking across different countries. Thus the innovation component tends to strengthen the overreaction/correction effect, while the long run equilibrium component tends to support the conventional effect. The cross-sectional forward discount regression has a less negative slope coefficient because of the variation across country pairs in the predictable component ($\bar{\mu}_0$) of the money growth differential, which strengthens the conventional effect.

6 Conclusion

This paper investigates the role of investor overconfidence in explaining the forward discount puzzle and predictability in the foreign exchange market. Our model shows that due to overconfidence, investors overreact to macroeconomic news, which leads to overshooting of both forward and spot exchange rates, with higher magnitude of overshooting in the forward rate than in the spot rate. Thus, the forward discount reflects the overreaction in the spot rate and predicts its subsequent correction. The forward discount bias results when this overreaction-correction effect dominates the conventional effect implied by uncovered interest rate parity.

In short-horizon time-series forward discount regressions, the overreaction-correction effect tends to dominate the conventional effect, resulting in the forward discount bias. In long-horizon time-series regressions, however, the traditional effect tends to grow in strength relative to the overreaction-correction effect, because mispricing in the spot exchange rate attenuates over time, whereas the effect of foreseeable differences in the expected growth rates of different currencies (differences which are recognizable without much use of subjective judgment) accumulates. Thus, the forward discount bias weakens in long-horizon time-series regressions.

Similarly, the forward discount bias weakens in a cross-sectional regression setting. The foreseeable part of the inflation differential plays a bigger role in cross-sectional regressions, strengthening the conventional effect. In time series regressions (especially short-horizon regressions), the innovation component of the inflation differential plays a larger role, increasing the importance of overreaction-correction effects.

Our analysis accommodates but does not require violations of relative purchasing power parity (PPP). We show that the existence of short-run violation of PPP by itself does not produce the forward discount bias under plausible parameter values. It can, however, amplify the overreaction-correction effect associated with investor overconfidence, thereby making the forward discount bias more pronounced.

Our analysis suggests some broader directions for research. Violations of Purchasing Power Parity in the form of overshooting of exchange rates relative to price levels is usually taken as exogenous in theoretical models. Such overshooting is not required for our main results, and therefore we do not explore its underpinnings. However, several considerations suggest that overconfidence can be a source of deviations from PPP. If there is a degree of market segmentation in which the prices of goods and services are influenced by the inflation expectations of participants in goods markets, whereas exchange rates reflect expectations of the currency traders, then the greater overconfidence among currency traders will tend to create greater overreaction in exchange rates than in the price levels for goods and services.

In foreign exchange markets, less than five percent of the transactions involves importers, exporters and other non-financial companies. Trading is dominated by institutional investors such as bank traders and hedge funds, who generally deal heavily with derivatives and speculate rather than hedge (Frankel and Rose 1995). Currency speculators are in their business precisely because they believe they have superior talents at forecasting changes in exchange rates. In contrast, most participants in the real goods markets including consumers are not primarily in the business of forecasting exchange rates. Therefore, currency traders are likely to be more overconfident about forecasting exchange rate than participants in the goods markets are about forecasting inflation. Furthermore, since currency prices are highly volatile, investors of all sorts are likely to receive very noisy feedback about their abilities to forecast exchange rate movements; psychological evidence suggests that such noisy feedback tends to contribute to overconfidence.

Another interesting direction for extension of our approach is to the term structure of domestic interest rates. The bond pricing literature has provided findings that are in some ways analogous to the international forward discount puzzle (e.g., Fama and Bliss, 1987; Campbell and Shiller, 1991; Cochrane, 1999; Bekaert and Hodrick, 2001; Cochrane and Piazzesi, 2005). A regression of the change in short term yields on the short-term forward-spot spread (difference between the forward interest rates and the short-term spot interest rate) gives a slope coefficient near zero and even negative, indicating the failure of the expectations hypothesis in the short horizons (the hypothesis predicts a slope coefficient of unity). The forward-spot spread also positively predicts holding period returns of long term bonds. Bekaert and Hodrick (2001) observe that researchers have had surprisingly little success explaining the empirical failure of the expectations hypothesis in terms of rational risk premia. It will be interesting to see whether overconfidence can offer an integrated explanation for these findings as well.

Appendix: Proofs

Proof of Lemma 1: First look at $z_t = \triangle \bar{\pi}_t^e = \mathrm{E}_t[\bar{\pi}_{t+1}] - \mathrm{E}_{t-1}[\bar{\pi}_t]$. By (5),

$$z_t = \mathrm{E}_t[\bar{\mu}_{t+1} + \alpha z_{t+1}] - \mathrm{E}_{t-1}[\bar{\mu}_t + \alpha z_t] \tag{28}$$

Taking expectation of both side of (28) with respect to date $t-1$ information set, we obtain

$$\mathrm{E}_{t-1}[z_t] = \mathrm{E}_{t-1}[\bar{\mu}_{t+1} - \bar{\mu}_t] + \alpha \mathrm{E}_{t-1}[z_{t+1} - z_t] \tag{29}$$

Similarly,

$$z_{t+1} = \mathrm{E}_{t+1}[\bar{\mu}_{t+2} + \alpha z_{t+2}] - \mathrm{E}_t[\bar{\mu}_{t+1} + \alpha z_{t+1}] \tag{30}$$

Taking expectation of both side of (30) with respect to date $t-1$ information set, we obtain

$$\mathrm{E}_{t-1}[z_{t+1}] = \mathrm{E}_{t-1}[\bar{\mu}_{t+2} - \bar{\mu}_{t+1}] + \alpha \mathrm{E}_{t-1}[z_{t+2} - z_{t+1}] \tag{31}$$

Subtracting (29) from (31), and combining terms,

$$\mathrm{E}_{t-1}[z_{t+1} - z_t] = \left(\frac{1}{1+\alpha}\right) \mathrm{E}_{t-1}[\bar{\mu}_{t+2} - 2\bar{\mu}_{t+1} + \bar{\mu}_t] + \left(\frac{\alpha}{1+\alpha}\right) \mathrm{E}_{t-1}[z_{t+2} - z_{t+1}] \tag{32}$$

A similar equation holds for $\mathrm{E}_{t-1}[z_{t+2} - z_{t+1}]$:

$$\mathrm{E}_{t-1}[z_{t+2} - z_{t+1}] = \left(\frac{1}{1+\alpha}\right) \mathrm{E}_{t-1}[\bar{\mu}_{t+3} - 2\bar{\mu}_{t+2} + \bar{\mu}_{t+1}] + \left(\frac{\alpha}{1+\alpha}\right) \mathrm{E}_{t-1}[z_{t+3} - z_{t+2}] \tag{33}$$

Substituting (33) into (32) and iterating forward, we obtain (14).

By equations (28) and (14), we can write z_t in terms of expectations of future money growth differential

$$\begin{aligned} z_t &= \mathrm{E}_t[\bar{\mu}_{t+1}] - \mathrm{E}_{t-1}[\bar{\mu}_t] + \alpha \mathrm{E}_t[\bar{\mu}_{t+2} - \bar{\mu}_{t+1}] - \alpha \mathrm{E}_{t-1}[\bar{\mu}_{t+1} - \bar{\mu}_t] \\ &+ \sum_{k=0}^{\infty} \frac{\alpha^{k+1}}{(1+\alpha)^k} \left\{ \mathrm{E}_t[\bar{\mu}_{t+k+3} - 2\bar{\mu}_{t+k+2} + \bar{\mu}_{t+k+1}] - \mathrm{E}_{t-1}[\bar{\mu}_{t+k+2} - 2\bar{\mu}_{t+k+1} + \bar{\mu}_{t+k}] \right\} \end{aligned} \tag{34}$$

Next look at $y_t = \triangle s_t - \bar{\pi}_t$. By (6), y_t satisfies the following mean reverting dynamics

$$y_t = \alpha(\theta - 1) z_t - \kappa y_{t-1} \tag{35}$$

It is straightforward to derive (12) by iteratively applying (35). Equation (10) follows from definition of y_t and (5). Finally, equation (13) follows from (5) and (7).

Proof of Proposition 1: Applying Lemma 1 on date 2, we obtain

$$\begin{aligned} z_2 &= u - \lambda^C \sigma \\ \triangle s_2 &= \theta \bar{\mu}_2 + \alpha \theta z_2 - \kappa y_1 \end{aligned} \tag{36}$$

Proposition 1 follows immediately upon substituting

$$u = \lambda^R \sigma - \delta = (1-\gamma)\lambda^C \sigma - \delta$$

(which follows from (20)) into (36) and then substituting $d_1 = \bar{\mu}_0 + \lambda^C \sigma$.

Proof of Proposition 3:

Let the exchange rate on date 0 be s_0. Since $\bar{\mu}_0$ and s_0 are constants, their values will not affect the correlations of the variables we consider here. Therefore, for simplicity we set $\bar{\mu}_0 = s_0 = 0$. On date 1, upon receiving the signal σ, the inflation differential ($\bar{\pi}_1$), the change in the spot rate ($\triangle s_1$), the spot rate (s_1), the forward rate (f_1), and the forward discount (d_1) are as follows.

$$\begin{aligned} \bar{\pi}_1 &= \eta_1 + \alpha\lambda^C \sigma \\ \triangle s_1 &= \eta_1 + \alpha\lambda^C \sigma \\ s_1 &= s_0 + \triangle s_1 = \eta_1 + \alpha\lambda^C \sigma, \\ f_1 &= s_1 + d_1 = \eta_1 + (1+\alpha)\lambda^C \sigma \\ d_1 &= \lambda^C \sigma. \end{aligned}$$

All the regressors, except d_1, contain η_1, the random realization of the money growth differential on date 1. $\bar{\pi}_1, \triangle s_1, s_1$ and f_1 can all be written in the same form of

$$\omega = b d_1 + c \eta_1,$$

for some positive constants b and c.

The R^2 of regressing $\triangle s_2$ onto a regressor of the form ω, denoted by R^2_ω, is

$$R^2_\omega = \frac{\text{cov}(\triangle s_2, \omega)^2}{\text{var}(\triangle s_2)\text{var}(\omega)}.$$

By Proposition 1,

$$\triangle s_2 = \beta_1 d_1 + v_2,$$

where

$$v_2 = \eta_2 - (1+\alpha)\delta.$$

Note that η_1, η_2, and δ are uncorrelated with each other. Further, they are all uncorrelated with the date 1 signal σ. It follows that

$$\text{cov}(\triangle s_2, \omega) = b\beta_1 \text{var}(d_1),$$

and

$$\text{var}(\omega) = b^2 \text{var}(d_1) + c^2 \text{var}(\eta_1).$$

Thus,

$$R^2_\omega = \frac{[b\beta_1 \text{var}(d_1)]^2}{\text{var}(\triangle s_2)[b^2 \text{var}(d_1) + c^2 \text{var}(\eta_1)]}.$$

Similarly, the R^2 of regressing $\triangle s_2$ onto the forward discount d_1, denoted by $R^2_{d_1}$, is given by

$$R^2_{d_1} = \frac{\beta_1^2 \text{var}(d_1)}{\text{var}(\triangle s_2)}.$$

It follows that the difference in predictive power as measured by $R^2_{d_1} - R^2_\omega$ is

$$R^2_{d_1} - R^2_\omega = \frac{\beta_1^2 c^2 \text{var}(\eta_1)\text{var}(d_1)}{\text{var}(\triangle s_2)[b^2\text{var}(d_1) + c^2\text{var}(\eta_1)]} > 0.$$

Proof of Proposition 4: Let $f_{1,3}$ denote the two period forward exchange rate, and $d_{1,3} = f_{1,3} - s_1$ be the two period forward discount. By a standard arbitrage argument and the assumptions in Section 4.4, we derive an equation similar to (7):

$$d_{1,3} = \text{E}_1[\bar{\pi}_3 + \bar{\pi}_2].$$

By equations (5), (8), and (16),

$$d_{1,3} = 2(\bar{\mu}_0 + \lambda^C\sigma) + \alpha\text{E}_1[\bar{\pi}_3^e - \bar{\pi}_1^e].$$

Applying Lemma 1,

$$\begin{aligned} \bar{\pi}_1^e &= \bar{\mu}_0 + \lambda^C\sigma \\ \bar{\pi}_3^e &= \bar{\mu}_0 + u. \end{aligned}$$

Thus,

$$\text{E}_1[\bar{\pi}_3^e - \bar{\pi}_1^e] = \text{E}_1[u - \lambda^C\sigma] = 0,$$

and so

$$d_{1,3} = 2(\bar{\mu}_0 + \lambda^C\sigma) = 2d_1.$$

This, combined with the fact that the two period change in the spot exchange rate $s_3 - s_1 = \triangle s_3 + \triangle s_2$, implies that the slope coefficient of $s_3 - s_1$ on $d_{1,3}$ is the average of the slope coefficient β_1 of $s_2 - s_1$ on d_1 and slope coefficient $\beta_{1,2}$ of $s_3 - s_2$ on d_1. By Lemma 1,

$$\triangle s_3 = \theta\bar{\pi}_3 - \kappa(\theta - 1)\bar{\pi}_2 + \kappa^2(\theta - 1)\bar{\pi}_1.$$

$\beta_{1,2}$ can be obtained by using $\bar{\pi}_t = \bar{\mu}_t + \alpha z_t$ and substituting $z_1 = \lambda^C\sigma$, $z_2 = u - \lambda^C\sigma$, $z_3 = 0$ and $u = (1-\gamma)\lambda^C\sigma - \delta$:

$$\beta_{1,2} = [\theta - \kappa(\theta - 1)(1 + \alpha)](1 - \gamma) + \alpha\kappa(1 + \kappa)(\theta - 1).$$

It follows that

$$\beta_1 - \beta_1' = \frac{\beta_1 - \beta_{1,2}}{2} = -\frac{1}{2}\left[\alpha\theta\gamma + \kappa\gamma(\theta - 1)(1 + \alpha) + \kappa(\theta - 1)(\alpha + \alpha\kappa - 1)\right],$$

so $\beta_1' - \beta_1$ is increasing with the degree of overconfidence (measured by γ). The slope coefficient in short-term forward discount regression is more negative than that in long-term forward discount regression when there is sufficiently high degree of overconfidence:

$$\beta_1 - \beta_1' < 0 \ \text{ if } \ \gamma > \frac{\kappa(\theta - 1)(1 - \alpha - \alpha\kappa)}{\alpha\theta + \kappa(1 + \alpha)(\theta - 1)}.$$

In particular, $\beta_1 < \beta_1'$ holds for all level of overconfidence if $\theta = 1$.

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Figure 1: **Overreaction and Correction of Exchange Rates**

This graph illustrates the expected path of movement for the spot and forward exchange rates from date 0 to date 2, conditional on a date-1 signal about date-2 money growth differential. The upper half of the figure depicts the case of a positive signal σ about money growth differential innovation. In response to a positive shock σ, the spot and the forward exchange rates increase to s_1 and f_1. They both overreact to σ: $s_1 > s_1^R; f_1 > f_1^R$, where the superscript R denotes the rational case without overconfidence-induced overreaction. The magnitude of overreaction is higher for the forward rate. After date-2 money growth differential is realized, the overreaction is on average corrected. The lower half of the figure depicts the case of a negative shock $-\sigma$.