Shared Appreciation Mortgages: Lessons from the UK*

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Abstract

The recent rise in shared appreciation mortgage (SAM) availability motivates careful consideration of underlying borrower incentives. The lender's share of appreciation in SAMs (*share*) is essentially a dynamic prepayment penalty imposed on the borrower. However, the borrower faces a moral hazard due to his ability to affect the penalty by reducing maintenance. We adapt a competing risks mortgage-pricing model to calculate SAM theoretical equilibrium rates. Our borrower possesses rational expectations of both the house price market and interest rates. Our simulation results may help explain the lack of secondary market interest for the UK SAMs containing extreme contract terms.

JEL G21

shared appreciation, mortgage pricing, options, prepayment penalty, default

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1. Introduction

One of the most innovative mortgage designs of the past decade is the shared appreciation mortgage (SAM). While the current variation of the is being originated by National Commerce Bancorp of Memphis throughout the Southeast in the United States, the SAM has been tried in both the U.S. and in Europe for over twenty years. The SAM has never been a popular alternative mortgage design, but periodically reappears like a mortgage version of a Phoenix arising from the ashes.

The SAM is targeted towards households that desire to either (1) buy a higher-priced house for the same monthly payment as that of a fixed-rate mortgage (FRM) on a lower priced house or (2) reduce their monthly payment compared to a FRM for the same-priced house. The financial institution lends the household a certain percentage of the appraised value of the dwelling at a below-FRM mortgage rate. In exchange, the household agrees to pay the financial institution a fixed percentage of the gain in property value (*e.g.,* 50% of any appreciation in property value) at (1) the end of the mortgage term, (2) if the loan is refinanced, or (3) if the property is sold. Table 1 provides an example of the types of SAM-FRM comparisons typically presented in newspaper and magazine articles about the benefits and costs of SAMs.

In this paper, we analyze the pricing-related issues of this type of SAM. The main contribution of this paper is the adaptation of a FRM option-pricing model for SAM pricing. In particular, we introduce the idea of pricing the lender's share of appreciation as a dynamic prepayment penalty and consider the borrower’s moral hazard problem of under investing in the house (i.e., delaying or foregoing maintenance). Our initial model demonstrates how the equilibrium contract rate and the value of the prepayment and default options change with changes in appreciation share and the term structure environment.

One goal of this paper is to raise questions about aspects of SAMs that could lead to their failure in the marketplace, from either the consumer side or supply side. What unforeseen risks exist for the purchasers of SAMs? Would an individual really give up expected appreciation to save on a monthly payment or purchase a bigger house? Therefore, another contribution of this paper includes proposals of ways to adjust the pure option theoretic SAM model for certain other problems that may arise. These problems are the moral hazard problem of underinvestment, the local self-selection problem of using national assumptions, and verifiability (or appraisal) concerns.

In addition, the pricing model presented in this paper may also be useful for pricing other versions of SAMs. Over approximately the last 10 years the UK has experimented with SAMs. The zero-coupon UK SAM is often an alternative to the reverse annuity mortgage (RAM) where the homeowner receives the entire amount of the loan at the time of origination and the homeowner loan liability is limited to amount borrowed plus a share of appreciation. There is no default risk for the UK SAM lender until loan termination since there is no payment due until loan termination, the household has no incentive to default; the same cannot be said for the RAM lender.

The remainder of the paper is organized as follows. Section 2 discusses an actual pool of UK SAMs, lessons from the UK SAMs, and recent news surrounding UK SAMs. Section 3 adapts a fixed-rate mortgage model for SAMs and presents the rational SAM borrower's decision equations. Section 4 discusses the results of our SAM equilibrium simulations under a variety of economic conditions, modifies the house price process to account for the pricing effects of underinvestment, and simulates the impact of increased house price volatility. Section 5 concludes the paper and offers extensions.

2. The UK Experience

In 1997 and 1998, SAMs issued by UK banks had either zero coupon or coupon. The extreme case of the zero-coupon UK SAM requires no payments due until the borrower sells the dwelling or dies; the maximum loan-to-value (LTV) that could be obtained on the zero-coupon UK SAM was 25%, based on appraised value of the dwelling at origination. In this maximum loan-to-value case, the appreciation ratio of 3 gives the lender 75% (calculated as LTV x appreciation ratio) of any property appreciation upon sale or prepayment. The 5.75% coupon UK SAM allowed borrowers to obtain up to 75% LTV in exchange for a percentage of any appreciation equal to LTV (an appreciation ratio of 1). None of the UK SAMs we study contain Mortgage Indemnity Guarantees (MIGs) where the lender must share in the process of loss coverage (see, Azevedo-Pereira et al., 2002).

2.1. UK SAMs and Mortality Tables

An attribute of the UK SAM is that an interesting separating equilibrium results: the typical profile of the UK SAM borrower is a household that is older in age (having paid-off their mortgage). As a consequence, mortality tables represent an important factor for repayment. Other variables that are potentially important for UK SAM prepayment are declines in health (the decision to move into a healthcare facility), change in location for retirement (the decision to retire in a warmer climate) and interest rate changes. We examine the prepayment history of a non-coupon UK SAM deal originated by one the banks in the UK. The deal was originated in July 1997 for zero coupon UK SAMs with a maximum of 25% LTV.

The loan pool underlying the UK SAM zero-coupon bond began with 1,340 loans. The average age of the first borrower on the UK SAMs is 68.69 years old with a standard deviation of 8.27 years (see Table 2, column A). During the first twenty-seven months, only 56 loans prepaid (see also Figure 1 for the prepayment distribution by month). This amount is considerably lower than prepayment rates on other types of mortgages such home equity loans and 125 LTV loans. Of the 56 loans that prepaid, only 18 were the direct result of death. However, it should be noted that 72 of the first borrowers died during the twenty-seven months; in many cases, there is a second borrower and the loan is not required to be repaid unless both the first and second borrower die. In numerous cases, the first and only borrower has died and the loan has not been recorded (since there is an average of 6 months from the time of death until the loan is considered prepaid).

The fact that only 18 of the 56 loans prepaid are due to death of both the primary and secondary borrower does not indicate that mortality is a poor explanatory variable for prepayment. Declining health of one or both of the borrowers can lead to a prepayment (e.g., moving to a relative's dwelling or assisted-living center). In addition, prepayment can be the result of the desired change in location for non-health reasons (e.g., moving from Scotland to Spain).

Even though only 4.18% of the UK SAMs prepaid after two years, the average annual return on the SAM loan was 23.21%. However, the standard deviation of the return is 21.12%. The annual appreciation ranged from 0% to 88%. Apparently, appraisal error is a concern since there was one loan that was repaid after three weeks for an annualized return to the lender in excess of 88%.

The coupon UK SAM prepayment information can be found in Table 2, column B. The number of prepayments (as a percentage of loans) is faster on the coupon UK SAM. This is not surprising given the greater incentive to refinance as interest rates drop. In addition, the number of prepayments owing to death is fewer than that of the zero-coupon UK SAM. Finally, the returns on the appreciation portion of the UK SAM are relatively higher. However, the problem of appraisal error seems to be present in the data as with the zero-coupon UK SAM.

More recent press has dealt harshly with the UK SAM. In the past few years, borrower groups have attempted to band together to raise funds to sue SAM lenders who hold very valuable options if the borrowers were to prepay. After the UK experienced extremely high house price appreciation from 1998 through 2002, SAM borrowers were often quoted using such statements as "ball-and-chain", "unable to move", and "lack of morality" when describing their SAM experience. All borrowers quoted had experienced dramatic house value appreciation with housing values usually doubling since origination.[1] Although we can find no evidence that such groups were successful in suing lenders, the "Banking Practices (Protection of the Elderly) Bill" that would have required banks to adopt and maintain specific practices in dealing with vulnerable elderly clients did have a first reading in the House of Commons on February 23, 2005. Yet it never reached a second reading.

In summary, the UK SAMs of the late 1990's represented an interesting alternative to the reverse annuity mortgage (RAM) for elderly homeowners. As expected, the lender's interest on the loan is highly volatile depending on the riskiness of the housing market as well as appraisal error. The average interest on UK SAMs ranged from 14 to 21% over the first two-years. The perplexing problem facing the UK SAM industry is the failure of secondary market investors to buy the UK SAM bonds even though the returns were initially quite high. We believe the SAM model presented in the following section can help in answering the questions surrounding failures of the UK SAMs.

[1] To reduce the extremely harsh penalties, more recent UK SAMs by Mortgage Express cap the total amount the borrower must pay in shared equity around 5% per year.

3. The Model

In order to explore these issues, we require several pricing features that are not found in the simple "what if" spreadsheet scenarios typically presented by lenders. Neither are the matrix pricing schedules for SAMs (based on, for example, a 10-year treasury) flexible enough since they do not consider borrower under investment, borrower self-selection, and other appraisal concerns. We require our pricing model to accommodate a borrower who is forward-looking and who makes rational decisions after considering potential future, "competing" default and prepayment options, given rational expectations of future house prices and interest rates.

In this paper we begin with the general fixed rate (FRM) mortgage model of Kau et al. (1992) and adapt the FRM pricing technique described in Hilliard et al. (1998) to form our SAM model. Using this backward pricing technique our SAMs model calculates equilibrium contract rates (given points) and endogenously determines the value of the prepayment and default options, given the level of the lender's *share*. Given any penalty (shared equity) that would be imposed by the level of the lender's *share* now or in the future, our rational borrower minimizes the value of the mortgage after evaluating the value of immediate or delayed option exercise

3.1. The Processes and the PDE

The processes assumed in the mortgage model of Kau et al. (1992) are well known and briefly presented here. Interest rates follow the square root process of Cox, Ingersoll, and Ross (1985):

$$dr = \kappa(\mu - r)dt + \sigma_r \sqrt{r} dz_r \tag{1}$$

where μ is the steady state mean rate, κ is the speed of adjustment factor, σ_r is the volatility of interest rates, and dz_r is a standard Wiener process. The property price process follows the standard stochastic process,

$$\frac{dH}{H} = (\alpha - s)dt + \sigma_H dz_H \tag{2}$$

where α is the total return to the property, s is the service flow, σ_H is the volatility of property returns, and dz_H is a Wiener process. We derive the risk-neutral house price process by changing the expected drift from α- s to α- s - λσ, where λis the market price of risk for house price. In a risk-neutral world, α-λσ is equal to the risk free rate of interest, yielding the process:

$$\frac{d\hat{H}}{\hat{H}} = (r - s)dt + \sigma_H dz_H \tag{3}$$

The second-order differential equation for any security X whose price is contingent upon the underlying variables in (1) and (3) is:

$$\frac{1}{2}H^2\sigma_H^2\frac{\partial^2 X}{\partial H^2} + \rho H\sqrt{r}\sigma_H\sigma_r\frac{\partial^2 X}{\partial H\partial r} + \frac{1}{2}r\sigma_r^2\frac{\partial^2 X}{\partial r^2} + \kappa(\mu - r)\frac{\partial X}{\partial r} + (r - s)H\frac{\partial X}{\partial H} + \frac{\partial X}{\partial t} - rX = 0 \tag{4}$$

where ρ is the correlation coefficient between dz_H and dz_r. The option values described in this paper can be obtained by imposing boundary conditions on (4).

3.2. The SAM Borrower's Decisions

In this section, we alter the borrower's decisions assumed in the fixed rate mortgage (FRM) to account for the different contractual obligations of a SAM. For the most part, we leave it to the reader to review the reasoning underlying the borrower decision equations in a standard FRM (our SAM equations were specifically adapted from Hilliard et al., 1998). We present the altered equations that contain the lender's share of appreciation (*share*).

With a SAM, the value of the mortgage, V, is no longer simply:

$V = PVRP - C - P$ (5)

as in the FRM case, but,

$V = PVRP - C(L) - P$ (6)

where *PVRP* is the present value of the remaining scheduled payments, *P* is the default (put) option, and *C(L)* is the prepayment (call) option, given *L* is any mandatory payment to the lender upon prepayment. Although the borrower still chooses to minimize the value of the mortgage, the SAM borrower no longer simply maximizes the combined values of *C* and *P* as in the FRM, but now must consider the potential effects of *L*. *L* can be viewed as a dynamic prepayment penalty, inseparable from prepayment, very similar to a transaction cost attached to the call option (see, Kau and Slawson 2002). With the addition of *L*, the value of *P* in the FRM does not equal the value of *P* in the SAM because of the competing nature of the options.2

The borrower decides among three courses of action immediately before each payment (each node in the binomial tree.) One, the borrower can exercise the call (prepay) by *purchasing the remaining payments* with the unpaid mortgage balance (*UMB*) and paying the lender's share. Two, the borrower can exercise the put (default) by, in essence, *selling the house* to the lender in exchange for elimination of all future payments. Three, the borrower can continue the mortgage by making the regularly scheduled mortgage payment.

In the case of prepayment the borrower's call option exercised and the lender's share is paid so that:

$C = PVRP - (UMB+L)$

$P = 0$

where $L = \max[share\ (H_t - H_0)\ , 0\ ]$, *share* is the percentage share of appreciation owed to lender according to the SAM contract, *Ht* is used to distinguish the current value of the house from value of the house at origination H_0, the value of the lender's share of appreciation. Using (6), in the case of prepayment, $V = UMB + L$.

In the case of default, the borrower's put option is exercised but the lender's share is forced to be exercised when it has no value (*e.g.* it expires worthless since the mortgage value enters default

2 We also note that *L* could be viewed as an option written by the borrower to the lender, where the lender exercises the option at the time of prepayment or at the end of the mortgage. In this

region of state space) so that:

$P = PVRP - H$

$C = 0$

where $L = 0$.

Using (6), $V = H$. Note that under current assumptions, in the absence of exogenous default, we preclude cases where the borrower would default with positive *share*. Since we assume LTV is less than 100% when t=0, then $(UMB+L) < Ht$ at all points in time since $L > 0$.

In the case of continuation none of the options are exercised so that:

$P = Pc$

$C = Cc$

where Pc is the present value (continuation value) of future put options and Cc is the present value of future call options, given the embedded cost of any expected lender share. Using (6), $V = PVRP - Cc - Pc$.

Therefore, the borrower's actions of minimizing the mortgage liability determines the value of the SAM as:

$$V = \min(UMB+L, H, PVRP-Cc-Pc) \quad (7)$$

It is interesting to note that, although the *share* paid by the borrower to the lender is a transaction cost to the borrower, and although this transaction cost does affect the borrower's decision to prepay, default, or continue, this transaction costs does *not* create a "wedge" between the value of the mortgage asset and the value of the mortgage liability. The reason for the lack of a wedge is because the cost to the borrower is a cent-for-cent benefit to the lender (or investor). Because the transaction cost is not external to the contract, but is instead an integral part of the SAM, the value of the mortgage asset is equal to the value of the mortgage liability when the lender and borrower have the same expectations.

This general equation (7) is also sufficient for the time immediately prior to the final payment when the lender's share option will be exercised or expire. However, again $PVRP = UMB = pmt$ and $Cc=Pc=0$, reducing (7) to $\min(pmt+L, H)$. In the case of default, as in the FRM case, P is

case, V = PVRP – C – P + L. Regardless of the view of L, the value of V remains the same.

set equal to (*pmt*-*H*) and $C = L = 0$. If the house has appreciated since origination, the lender receives *pmt*+*L*, and *L* is stored for use in earlier time steps as we backwards price within the lattice.

4. Results

4.1. Increasing Appreciation Share

The initial results are presented in Table 3. The house price parameters are σ_H= 15% and c = 8.5%. The interest rate parameters for all panels of the table are σ_r = 5%, κ= 25%, and μ= 8%. In the *upward*, *flat*, and *downward* term structure cases, r_1 is set to 3%, 7.5%, and 12%, respectively. Each of the 3 panels in Table 3 contains the calculated equilibrium rates, and values of *PVRP*, *C*, and *P* given SAM *share* values of 0% (equivalent to a FRM), 5%, 10%, 20%, 30%, 40%, 50%, and 75%. In each case, the model solves for the equilibrium contract rate for a discount of 1% and an initial LTV of 75% (similar to the maximum LTV of the coupon UK SAM). As expected, all SAM equilibrium contract rates are lower than their FRM (0% *share*) counterparts in the same economic environment. For each environment, as *share* increases (equivalent to a rising prepayment penalty) there is little effect on the equilibrium contract rates and option components for increases in *share* above 20% (see figures 2, 3, and 4). Increases in *share* (increasing the current prepayment penalty) above approximately 20% does not have a dramatic effect on components because the borrower is not expected to prepay in cases where house value is sufficiently above house value at origination. This is because with higher future house prices, the penalty reaches a point where additional penalty (higher *share*) does not affect the borrower's decision to continue the mortgage and avoid the penalty. On the opposite end of the house price spectrum, when future house values are below the initial house value ($0 appreciation to share), higher levels of contracted *share* (higher stated percentage penalties) have no effect on the value of immediate prepayment or default. Yet, the call option does not lose all of its value because there still remains numerous situations where prepayment may be optimal in the future when there is no penalty (Ht < Ho) or very little penalty.

4.2. The Moral Hazard of Underinvestment

One of the major concerns with the shared appreciation mortgage is the problem of underinvestment (see Miceli and Sirmans [1994] for a discussion of maintenance risk on RAMs). Since the borrower is forgoing a percentage of appreciation in the dwelling in exchange for a lower coupon rate, there is the concern that the borrower will not maintain the dwelling to the lender's satisfaction. A typical example of the underinvestment problem is maintenance of the dwelling's roof. With a standard fixed-rate mortgage, the borrower is likely to repair or reroof the dwelling at some point increasing the value of the dwelling relative to not having the roof repaired. With a SAM, the less initial costs the borrower recaptures, the less likely that the roof will be repaired (assuming that the borrower does not get sufficient utility from a new roof) leading to a decrease in the property's value.

We propose that the level of underinvestment can be modeled by simply altering the underlying house price (2) to

$$\frac{d\hat{H}}{\hat{H}} = (r - s - u)dt + \sigma_H dz_H \qquad (9)$$

where u is the level of underinvestment. One could equate this "underinvestment" to a borrower paying to himself the maintenance cost equivalent to u rather than repairing the house. Therefore, this level of underinvestment is treated as an additional dividend, reducing the total expected return in the house price equation.

Table 4 expands on the flat term structure scenario in Table 3 by including columns for levels of underinvestment equal to 0% (no underinvestment), 1%, 2%, 3%, 4%, and 5%. For any level of *share* (reading across any row) the equilibrium contract rate increases with higher levels of underinvestment, *u*. For example, a borrower sharing 75% of the appreciation with the lender, would receive a rate of 8.768% if a normal level of maintenance is expected to occur. In the absence of asymmetric information, if the borrower is expected to underinvest (through either ignoring maintenance or consuming the house) at a rate of 5% per year, the borrower would receive a rate of 10.288%. This is due to increases in both the borrower's prepayment and default options. The prepayment option is more valuable because with less appreciation expected in the house due to reduced maintenance, any future prepayment penalty is reduced. At the same time, the reduced maintenance increases the chance that the house will decline in value so there are more expected instances of default.

If we allow for asymmetric information, the borrower's moral hazard creates a valuation problem for the lender. From Table 4, if the lender shares 75% and expects the increased share to cause the borrower to underinvest 1%, the lender offers a contract rate of 8.983% and 1 point for a par value of 0.9900. If, however, the borrower's actual level of underinvestment is 5%, using 8.983% as the contract rate, our model produces a par value of the mortgage of 0.9330, a reduction in value of 5.75%. The SAM lender who underestimates an individual borrower's actual reduction in maintenance will lend more than the mortgage is worth.

In order to rectify the underinvestment problem, a number of solutions have been posited. First, the borrower can be credited for all capital improvements simply by maintaining records of payment; the base amount of the loan is then raised by the amount of the improvements. Unfortunately, this approach suffers from two problems. The first problem is that it is possible to obtain records documenting improvements that did not actually occur or that were in excess of the actual amount paid for the improvement. The second problem is the crediting of improvements that do not contribute dollar-for-dollar with an increase in property value.

The second approach for rectifying the underinvestment problem is to have an appraiser estimate the percentage of property value increase that is attributed to capital improvements. While this has the advantage of minimizing the problems associated with the first approach, the second approach introduces the problem of appraisal error – the ability of the appraiser to actually determine the correct percentage of appreciation owing to capital improvements.

4.3. Self-Selection

Another potential concern for the SAM as a nationally available contract is self-selection. Originators of SAMs must adhere to specific guidelines before the mortgages will be purchased in the secondary market. Those setting prices in the secondary market must develop composite expectations for the properties underlying the pool of mortgages. A SAM will be overpriced if an individual obtains a lower rate SAM but has local market information about below average appreciation. In other words, those most likely to obtain a SAM are those in a declining or flat housing market, leading potentially to overpricing in the secondary market if no adjustment is made to price for the possibility of self-selection in local markets. However, if the appreciation component in a SAM could be more directly linked to a local index (see Shiller and Weiss, 2000), rather than the actual change in the home's appraised value, some of the self-selection problem could be alleviated.

Our model is able to evaluate self-selection by considering the price effects of a reduction in the perceived house price volatility. Table 5 presents the results of increases in asset volatility for 0%, 10%, 20%, 30%, 40%, and 50% appreciation share SAMs in Panel 2 of Table 3. In every case, as the level of asset volatility increases from 5% to 40%, the equilibrium contract rate increases.

This is because the possible higher future house prices will only deter the borrower from prepaying, with the higher penalty of *share*. Alternatively, the possible future lower house prices (due to the increased volatility) will not only encourage prepayment with lower penalties, but also increase the chances that the house value will fall enough to trigger default. These results show that under asymmetric information where the borrower perceives that the true house volatility is much lower than projected by a SAMs lender, such an individual would be more likely to select a SAM at the expense of the lender (and secondary market participants).

Also in Table 5, we notice that for higher levels of σ_H increasing *share* reduces the equilibrium contract rate more than with lower levels of σ_H. For σ_H = 5%, as *share* increases from 10% to 50%, the equilibrium contract only decreases less than 7.1 basis points from 8.306% to 8.235%. For σ_H = 40%, the same increase in *share* decreases the rate 245.8 basis points from 15.617% to 13.159%. The reason for the difference can be determined when we examine the individual option values. In the σ_H = 5% case the prepayment option value decreases slightly with virtually no effect on the default option. Whereas, in the σ_H = 40%, the prepayment option loses roughly 2/3rds of its value from 0.54363 to 0.18586, while the default option only increases from 0.28145 to 0.36956.

4.4. Verifiability Issues

A third aspect of the SAM is the problem of appraisal error. Like most mortgage loans, the underlying property is appraised when the loan is originated. Unlike most mortgage loans, the appraiser's estimate of market value plays a pivotal role in the amount repaid to the lender. For example, a UK SAM loan was made on a dwelling that was appraised at £130,000. After two weeks, the loan was repaid and the house was revalued at £150,000. The borrower had to repay the loan amount (£18,000) plus 75% of the appreciation in value (£15,000) for a total loan repayment after two weeks of £33,000. Although there is a possibility that the housing price jumped by £20,000 in two weeks, it is more than likely that the problem was appraiser error at origination.

As with a SAM in the absence of appraisal error, a SAM in the presence of appraisal error does not create a "wedge" between the mortgage asset from the lender's perspective and the mortgage liability from the borrower's perspective unless there is asymmetric information. This is because the extra cost borne by the borrower at origination is transferred to the lender upon prepayment or at the end of the mortgage. In the case of asymmetric information, however, where either the borrower or lender (or investor) perceives the house has been mis-appraised at origination, we propose that our model can accommodate this appraisal error by making two adjustments to our model. First, the LTV ratio should be adjusted to the true LTV ratio at origination. Second, the lender's share in each case of prepayment should be changed from $L = \max[share\ (H_t - H_0)\ , 0\ ]$to $L = \max[share\ (H_t - H_{App})\ , 0\ ]$, where H_{App} is the appraised value of the house at origination and H_t remains the actual value of the house at time t (given the adjustment in LTV). In this way, a borrower or lender can estimate the effects on SAM price due to appraisal error.

5. Conclusions

Given the reintroduction of shared appreciation mortgages (SAMs) it is important to know what assumptions about borrower behavior and contractual provisions are necessary to justify more widespread acceptance by secondary market participants. Having a pricing model that simulates the tradeoff of options within a base case SAM, which is also flexible enough to accommodate other conditions or concerns, should aid in the understanding of basic SAM and SAMs with slight variations. Our paper provides a competing risks mortgage-pricing model that has been adapted to accommodate the lender's share within a SAM as a dynamic prepayment penalty. We demonstrate the flexibility of the model by offering ways in which the model can be tailored to other factors that may affect price.

Our general conclusions reveal that the actual contract rates on the July 1997 pool of UK SAMs appear low compared to what our pure option theoretic simulations suggest and may help explain the lack of secondary market interest for UK SAMs containing extreme contract terms. We also show that equilibrium contract rates are more sensitive to the moral hazard problem of deferred maintenance than changes the lender's *share* of appreciation. Specifically, since the lender's share is equivalent to a prepayment penalty, increases in the lender's contract *share* of appreciation above 20% - 30% has very little impact on the equilibrium contract rate and option values. Consistent with expectations for steep upward sloping term structure environments, our simulations show little incentive for the borrower to acquire a SAM. In an upward sloping term structure environment, because both future increase in rates and future increase in house prices make it less likely for the borrower to prepay for optimal endogenous reasons, a lender will not offer a lower contract rate.

Given the concerns of potential for appraisal error, the moral hazard of underinvestment, and the reduced likelihood of prepayment with higher share, SAMs as an investment should be good investments the more certain the appraisal, the more likely the borrower is maintaining the property, and the greater the chance that the borrower will prepay (or be forced to prepay) for exogenous reasons. Therefore, SAMs may be better suited for housing such as condominiums. A SAM for a condominium that requires common area maintenance fees should substantially reduce the risk associated with the first two concerns. Requiring settlement dates other than the full term of the mortgage could reduce the third concern.

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Table 1: Typical SAM vs FRM Comparison

	SAM (50% of appreciation)		FRM
Initial Home Value	$ 325,000	$ 325,000	$ 325,000
Contract Rate	7%	7%	8.50%
House Appreciation	2%	6%	N/A
Years in Home	7	7	7
Yr 7 Balance	$236,983	$236,983	$242,006
Total Monthly Pmts	145,302	145,302	167,931
Home Value at Payoff	373,323	488,680	N/A
Home Appreciation	48,323	168,680	N/A
Lender's Share	24,161	81,840	-
Loan Payoff	261,145	318,823	242,006
Total Paid to Lender	406,447	464,125	409,937

Source: URL: http://www.military.bankrate.com/mtry/news/mtg/20000907.asp

Table 2: Actual UK SAM deal description

Bond Issue Date July 1997

Original Loan Pool	(Column A) Zero-Coupon	(Column B) Coupon SAM
Number of loans	1340	326
Average coupon rate	0.00%	5.75%
Average share of appreciation	23.79%	56.49%
Average age of borrower	68.69	60.00
Std dev of borrower's age	8.27	8.27
Average dwelling (pounds)	136,889	151,643
Std dev of dwelling (pounds)	107,514	116,173
Average Loan (pounds)	31,532	83,610
Std dev of Loan (pounds)	21,917	68,076

Prepayment Information	Zero-Coupon	Coupon SAM
Cutoff date	Aug-99	Aug-99
Number of prepayments	56	36
% of pool that has prepaid	4.18%	10%
Average annual return on SAMs	23.31%	11.49%
Std dev of avg annual return	21.12%	14.27%
Average age of prepayers	68.82	62.13
Std dev of prepayer age	10.58	9.25
Deaths	72	9
Redemptions from death	18	4

This Table presents original loan pool and prepayment information for the first twenty-seven months of an actual pool of zero-coupon bond UK SAMs and coupon bond UK SAMs securitized in July 1997.

Table 3: SAM Equilibrium Contract Rates and Option Component Values

Panel 1: Upward sloping term structure (r1 = 3%)

share	Equilibrium Rate	Price	PVRP	Call	Put
0%	7.264%	0.9900	1.07873	0.02265	0.06606
5%	7.222%	0.9900	1.07428	0.01757	0.06668
10%	7.205%	0.9900	1.07238	0.01108	0.07128
15%	7.200%	0.9900	1.07181	0.00779	0.07402
20%	7.199%	0.9900	1.07175	0.00743	0.07428
30%	7.199%	0.9900	1.07168	0.00734	0.07433
40%	7.199%	0.9900	1.07168	0.00734	0.07433
50%	7.199%	0.9900	1.07168	0.00734	0.07433
75%	7.199%	0.9900	1.07168	0.00734	0.07433

Panel 2: Flatter term structure (r1 = 7.5%)

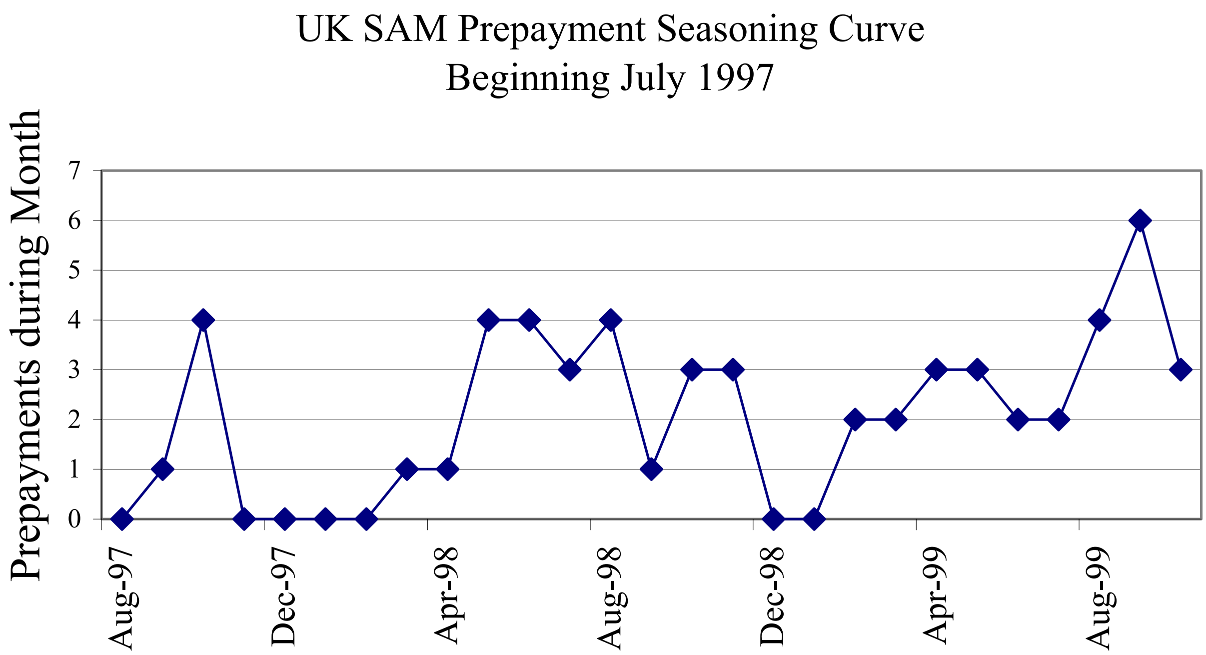
share	Equilibrium Rate	Price	PVRP	Call	Put
0%	9.129%	0.9900	1.12445	0.09138	0.04307
5%	8.965%	0.9900	1.10810	0.07851	0.03958
10%	8.862%	0.9900	1.09795	0.06803	0.03992
15%	8.805%	0.9900	1.09228	0.06207	0.04021
20%	8.781%	0.9900	1.08987	0.05948	0.04039
30%	8.768%	0.9900	1.08864	0.05810	0.04054
40%	8.768%	0.9900	1.08857	0.05804	0.04053
50%	8.768%	0.9900	1.08857	0.05804	0.04053
75%	8.768%	0.9900	1.08857	0.05804	0.04053

Panel 3: Downward sloping term structure (r1 = 12%)

share	Equilibrium Rate	Price	PVRP	Call	Put
0%	11.956%	0.9900	1.24205	0.22991	0.02214
5%	11.574%	0.9900	1.20850	0.19863	0.01987
10%	11.298%	0.9900	1.18112	0.17153	0.01959
15%	11.073%	0.9900	1.16048	0.15137	0.01910
20%	10.924%	0.9900	1.14680	0.13830	0.01850
30%	10.783%	0.9900	1.13392	0.12557	0.01835
40%	10.741%	0.9900	1.13017	0.12193	0.01824
50%	10.733%	0.9900	1.12943	0.12122	0.01821
75%	10.733%	0.9900	1.12937	0.12117	0.01821

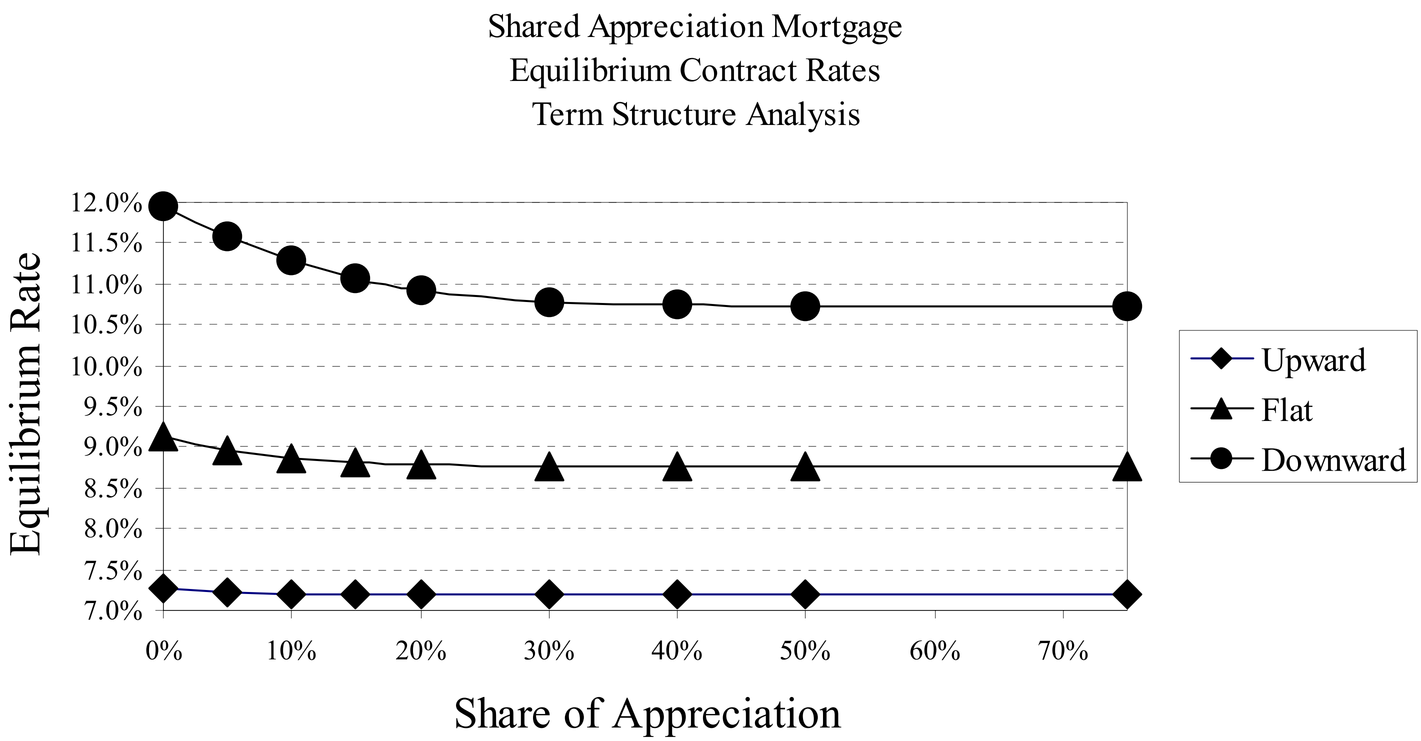
This table presents equilibrium contract rates, the present value of remaining regular payments, the value of the borrower's call option and the value of the borrower's put option for a SAM, given three different term structures as share varies from 0% (the fixed rate mortgage case) to 75% (the maximum allowed by the UK SAM. In each case the service flow is 8.5%, the interest rate standard deviation is 15%, the mean reversion coefficient is 25%, the steady spot rate is 8%, the loan to value ratio is 75%, and the term to maturity is 30 years. Given 1 point, the par value of the SAM is 0.9900 in each case.

FIGURE 1



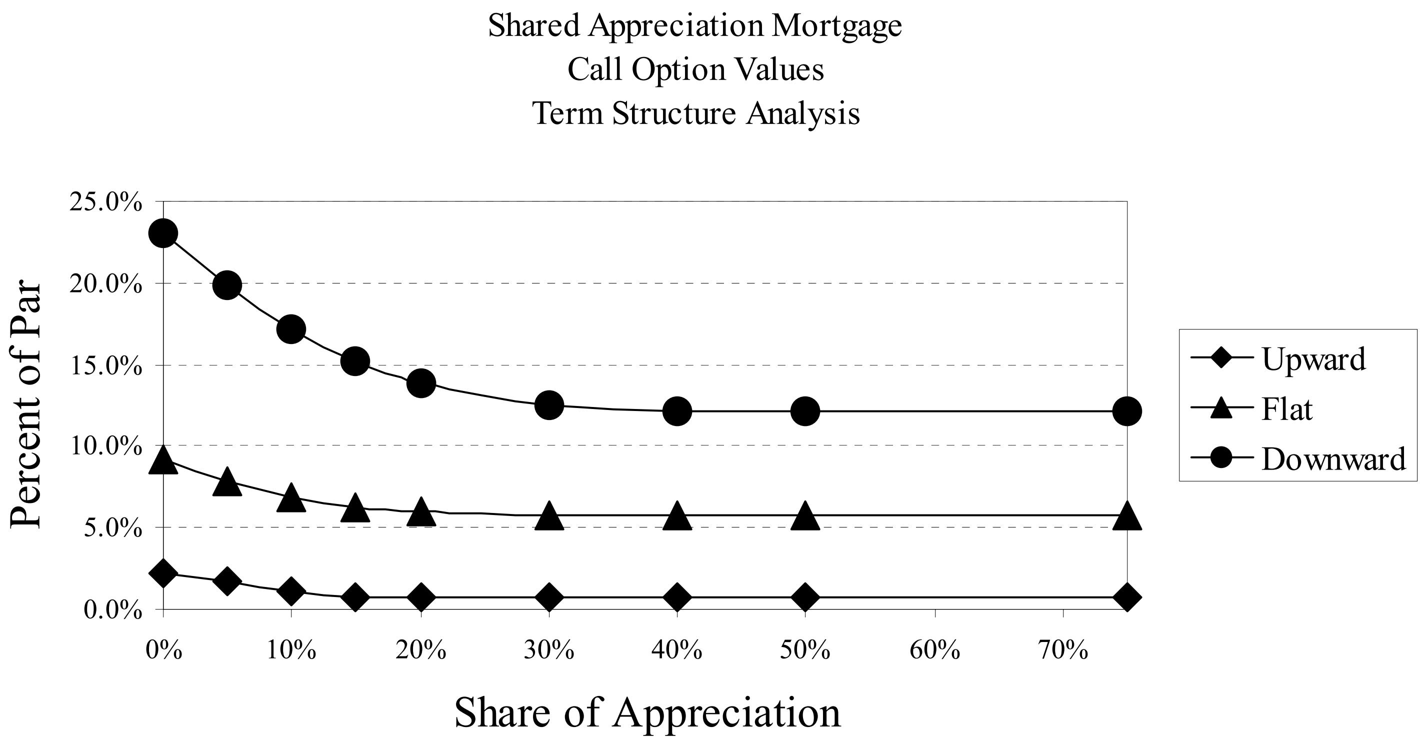
This figure represents the prepayment distribution by month for the first twenty-seven months of a pool of zero-coupon bond UK SAMs securitized in July 1997. During the period, only 56 of the 1,340 loans prepaid.

FIGURE 2



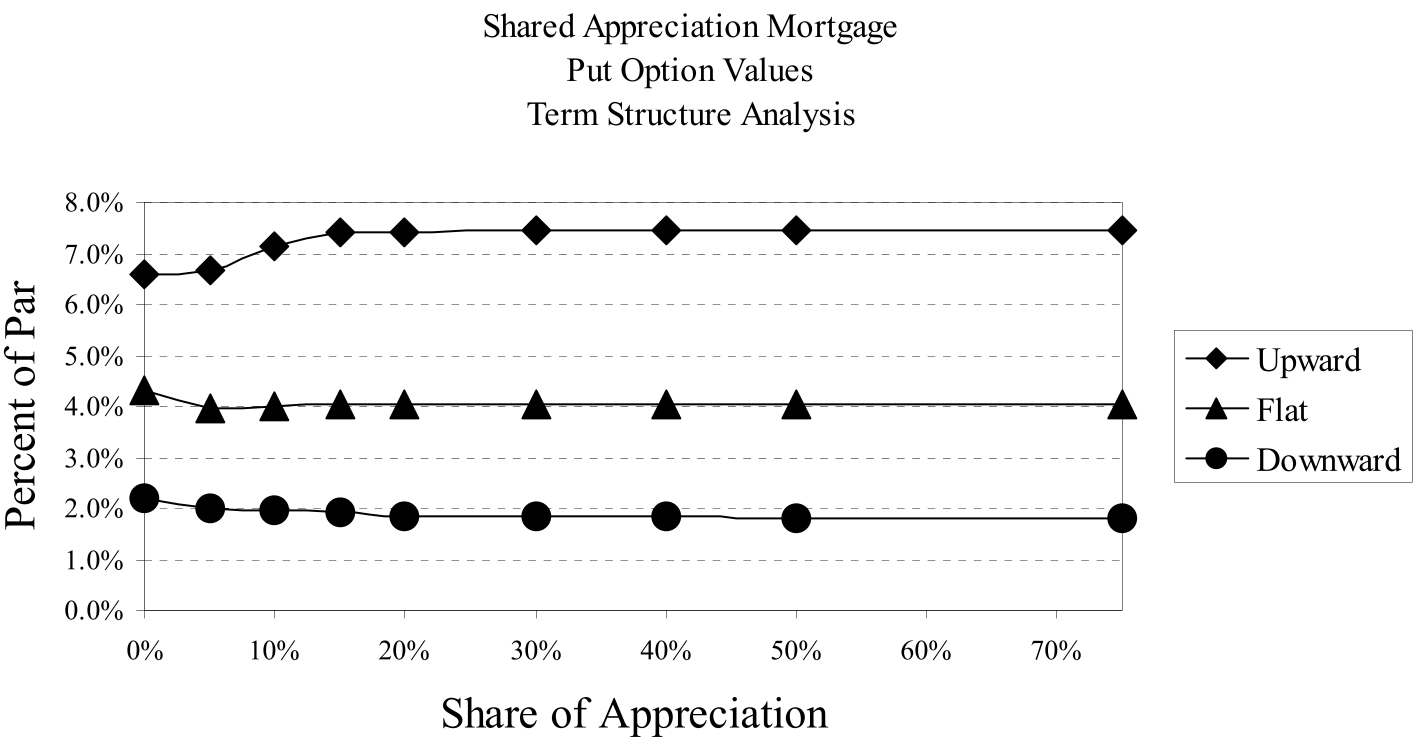
For this figure the term structure is relatively flat (7.5% spot interest rate, 8% steady state spot rate), the service flow is 8.5%, the house price standard deviation is 15%, the interest rate standard deviation is 15%, the mean reversion coefficient is 25%, the loan to value ratio is 75%, and the term to maturity is 30 years. Given 1 point, the par value of the SAM is 0.9900.

FIGURE 3



For this figure the term structure is relatively flat (7.5% spot interest rate, 8% steady state spot rate), the service flow is 8.5%, the house price standard deviation is 15%, the interest rate standard deviation is 15%, the mean reversion coefficient is 25%, the loan to value ratio is 75%, and the term to maturity is 30 years. Given 1 point, the par value of the SAM is 0.9900.

FIGURE 4



For this figure the term structure is relatively flat (7.5% spot interest rate, 8% steady state spot rate), the service flow is 8.5%, the house price standard deviation is 15%, the interest rate standard deviation is 15%, the mean reversion coefficient is 25%, the loan to value ratio is 75%, and the term to maturity is 30 years. Given 1 point, the par value of the SAM is 0.9900.

Table 4: SAMs Equilibrium Contract Rates given the Moral Hazard of Underinvestment

	Level of Underinvestment					
share	0%	1%	2%	3%	4%	5%
0%	9.129%	9.338%	9.585%	9.876%	10.205%	10.566%
5%	8.965%	9.187%	9.445%	9.744%	10.079%	10.451%
10%	8.862%	9.083%	9.342%	9.642%	9.981%	10.355%
15%	8.805%	9.024%	9.282%	9.577%	9.915%	10.286%
20%	8.781%	8.997%	9.252%	9.544%	9.879%	10.248%
30%	8.768%	8.983%	9.237%	9.529%	9.862%	10.229%
40%	8.768%	8.983%	9.237%	9.528%	9.861%	10.228%
50%	8.768%	8.983%	9.237%	9.528%	9.861%	10.228%
75%	8.768%	8.983%	9.237%	9.528%	9.861%	10.288%

This table presents equilibrium contract rates for a SAM, given changes in share from 0% (the fixed rate mortgage case) to 75% (the maximum allowed by the UK SAM) as level of underinvesetment varies from 0% to 5%. In each case the term structure is relatively flat (7.5% spot interest rate, 8% steady state spot rate), the service flow is 8.5%, the interest rate standard deviation is 15%, the mean reversion coefficient is 25%, the loan to value ratio is 75%, and the term to maturity is 30 years. Given 1 point, the par value of the SAM is 0.9900.

Table 5: SAM Sensitivity to House Volatility

	Level of *share*					
House Volatility	0%	10%	20%	30%	40%	50%
5%	8.388%	8.306%	8.265%	8.244%	8.237%	8.235%
10%	8.543	8.405	8.357	8.345	8.343	8.343
15%	9.129	8.862	8.781	8.768	8.768	8.768
20%	10.162	9.601	9.424	9.405	9.405	9.405
25%	11.568	10.589	10.196	10.161	10.161	10.161
30%	13.480	11.945	11.135	11.062	11.061	11.061
35%	15.613	13.607	12.203	12.050	12.049	12.049
40%	18.165	15.617	13.436	13.161	13.159	13.159

This table presents equilibrium contract rates for a SAM, given changes in house price volatility from 5%-40%, as level of *share* varies from 0% to 50%. In each case the term structure is relatively flat (7.5% spot interest rate, 8% steady state spot rate), the service flow is 8.5%, the interest rate standard deviation is 15%, the mean reversion coefficient is 25%, the loan to value ratio is 75%, and the term to maturity is 30 years. Given 1 point, the par value of the SAM is 0.9900.